

Implementing Numerical Linear Algebra in the Undergraduate Science and Engineering Curriculum

Ronald F. Taylor
Wright State University
Dayton, Ohio

NIU LA'09
LINEAR AND NUMERICAL LINEAR ALGEBRA: THEORY, METHODS, and APPLICATIONS
Conference at Northern Illinois University, DeKalb, Illinois

Department of Computer Science and Engineering



What This Talk Is and Is Not

- What it is ...
 - Teaching Observations : Books , Curriculum , Trends , and Challenges
 - Curriculum Observations: “Computational Science” and the Mathematics Component (Especially Computational Linear Algebra)
 - Therapy (maybe)
- What it is not ...
 - Technical Presentation on a Specific LA Topic
 - Solution to Dilemmas Teaching Computational Mathematics to Engineering Students

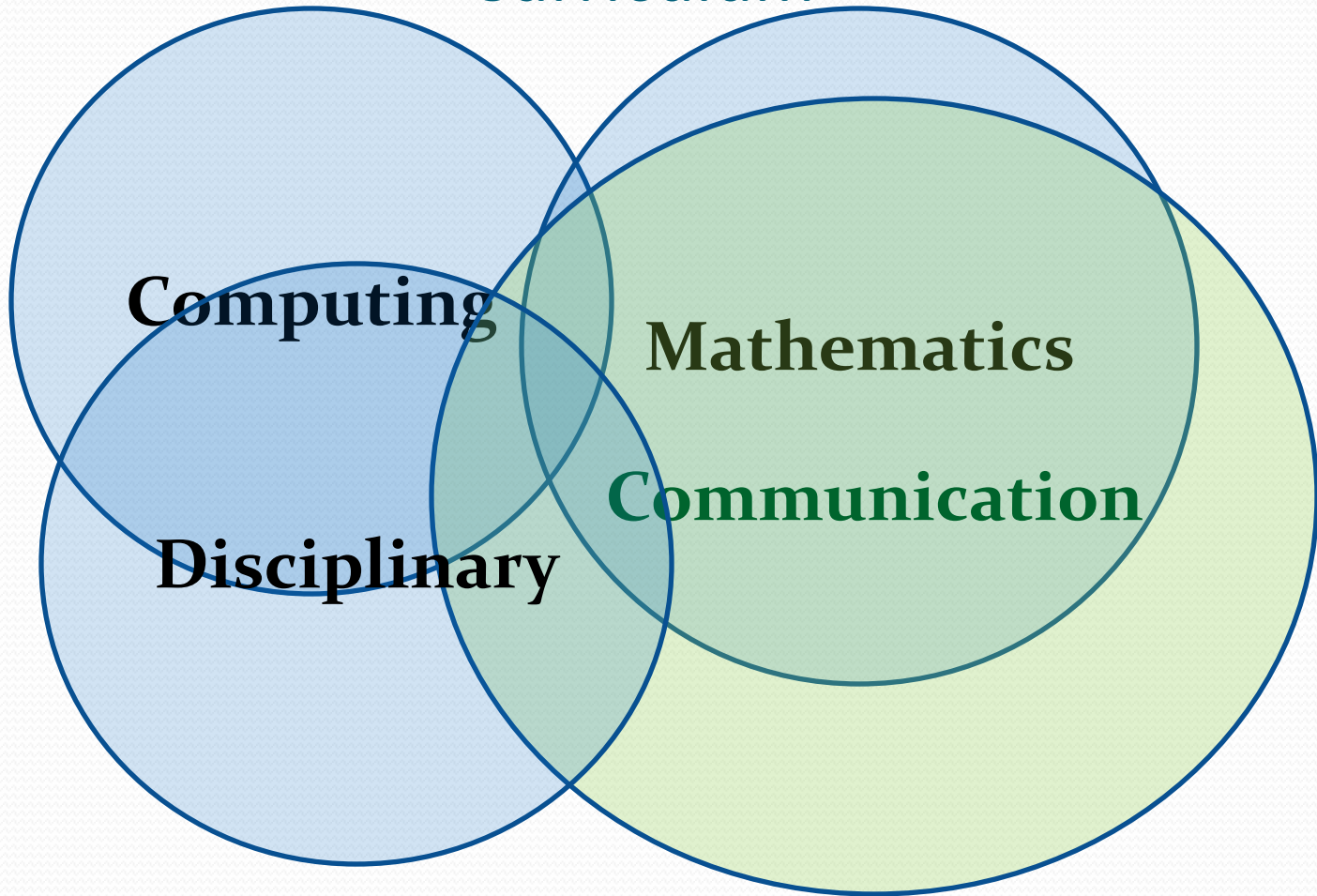
OUTLINE

- Curriculum and Courses
- Mathematics and Computing
- Textbooks and Manuals
- Computational Linear Algebra Modules



CURRICULUM AND COURSES

Mathematics – Especially Linear/Matrix Algebra: An Essential Interacting Component in the Curriculum



Importance of Communication in the Curriculum

- “Reading” (papers/texts) with Matrix Equations – Important for Beginning Grad Students
- Discussion of an Algorithm is Different from a Proof
- Numerical Examples and Test Matrices.
- Documentation – Insight Not Just Numbers!!!!!!
- To the Department, College, University and Beyond!

WSU DE Course With Linear Algebra

- Advantages
 - Easy to introduce applications in ODE
 - Shows central role of matrix eigenvalue problem
- Disadvantages
 - Too much material for one course.
 - Jumps from topic to topic with shallow learning
- Maybe “Linear Algebra with Differential Equations”!

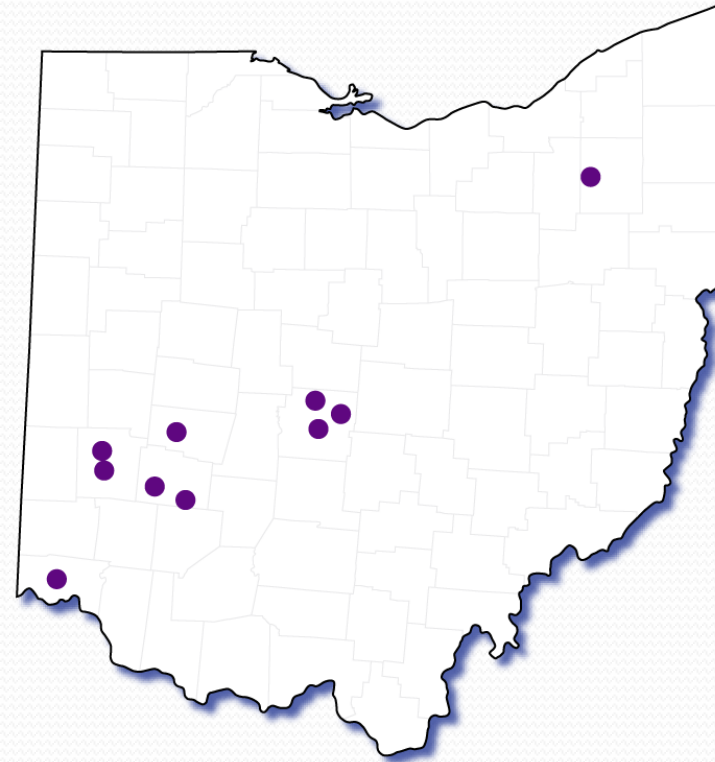
Key Curricular Issues (Local?)

- How and when should students learn programming?
- We are converting from quarters to semester-based courses . What new topics?
- Introduce a lab component to computational linear algebra course?
- How to teach computational linear algebra to students with weak linear algebra skills.

Undergraduate Minor Program

Ralph Regula School of Computational Science

- Funding from NSF
- Participating (2 and 4 yr):
 - Capital University
 - Central State University
 - Columbus State Community College
 - Sinclair Community College
 - Kent State University
 - The Ohio State University
 - University of Cincinnati
 - Wilberforce University
 - Wittenberg University
 - Wright State University
- Program started in Autumn 2007





MATHEMATICS AND COMPUTING

Applied or “Traditional” Mathematics?

- “Engineering Mathematics” or “Math for CSE”
 - WSU Computer Science Majors Start with Discrete Mathematics (Math Dept) – Then Calculus
 - WSU Mechanical Engineering Majors Start with “Engineering Math” (Engineering College)– “Just in Time Math” Approach - Then Calculus

“Just in Time” Mathematics

- Advantages
 - Courses can be coordinated
 - Improved math recall for application courses
- Disadvantages
 - Shallow “brittle” learning experience
 - Missing “the big picture”

Abstract or Computational Mathematics?

- **Good!!!:** “... appreciation ... comes with seeing some of the scaffolding. (Carl Meyer)”
- **Interesting!:** “... concentrate on the essential facts ... attention is not disturbed by unimportant details.” (Erwin Kreyszig)
- **Ouch!:** “The computer is important, but not to mathematics.” (Paul Halmos)

Getting Correct “Answers”: Use of Tools vs Languages

- “I just teach the freshman MATLAB! They like it!”
 - An Undergraduate Teaching Faculty Colleague
- “Help me start a programming course! My students know MATLAB but the sponsor wants C++ code”
 - A Graduate Research Faculty Colleague
- SIAM CSE09– “Billions of linear equations easily solved!”
(Question Not Asked: “Are the answers ‘correct’ ???”)



TEXTBOOKS AND MANUALS

Student-Friendly Texts – Examples

- Computational Science and Engineering, Strang
-- Modernized “Engineering Math”
- Introduction to Linear Algebra, Strang
-- Popular Text with Online Video Course
- Linear Algebra with Applications, Leon
-- “At Last Computer Exercises for Linear Algebra!”

Student Reaction Surprises!

Compare: Encyclopedic / Comprehensive

- Matrix Computations, 3rd ed, Golub & Van Loan
- Numerical Linear Algebra & Applications, Datta


With: Shorter Books with Shorter Chapters

- Numerical Linear Algebra, Trefethen & Bau
- Matrix Methods in Data Mining & Pattern Recognition, Eld'en – SIAM Fundamentals Of Algorithms Series

Educational Publication Ideas

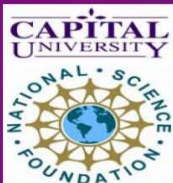
A Lab Manual of Matrix Computation

- Usable at Junior to Beginning Graduate Level
- Focus on Matrix Methods and Algorithms & Not Computing Hardware/Architecture Issues
- Integrated Lab and Lecture format
- Small one hour lab exercises – 8 per 10 week quarter/ 12 per 15 week semester
- Larger one week projects – 4 per 10 week quarter / 6 per 15 week semester
- Final project – Group option with presentation by students.
- Include a Supplement for Instructors



COMPUTATIONAL LINEAR ALGEBRA MODULES

CSE Modules – Four Year NSF Grant Capital University – Lead Institution



Development and Dissemination of Computational Science Educational Materials and Curricula at the Undergraduate Level

Funded by the National Science Foundation (DUE 0618252)

With Capital University as the lead institution, this project establishes a consortium of 35 faculty from 16 institutions representing major STEM (science, technology, engineering, mathematics) disciplines with a common agenda to develop and disseminate computational science educational materials suitable for the undergraduate arena. Over the four year period from 2007-2010, this \$480,000 grant enhances the education of undergraduate students at community colleges, liberal arts and comprehensive institutions as well as research one institutions.



- Terry Lahm, PI, Geology and Environmental Science
- Roxanna Postolache, Economics
- David Reed, Computer Science
- Karl Romstedt, Biology
- Alan Stam, Ecology
- Roy Roychoudhury, Finance



- Eric Stalberg, Computer Science



- Amil Anderson, Chemistry
- Margaret Goodman, Biology
- James Noyes, Computer Science
- John Ritter, Geology



- Gregory Baker, Mathematics



- Susan Sakimoto, Geology



- Eric Grosflis, Geology
- Linda Reinen, Geology



- Nuh Aydin, Mathematics
- Sheryl Hemkin, co-PI, Chemistry
- Judy Holdener, Mathematics
- Andrew Kerkhoff, Biology
- Tim Sullivan, Physics



- Kent Fisher, Physics
- Morteza Javadi, Biology
- Gerald Mueller, co-PI, Mathematics



- Ignatios Vakalis, co-PI, Computer Science



- Helen Pointkivska, Biology



OTTERBEIN
COLLEGE

- David Robertson, Physics



- Luis Fernandez, Economics
- John Petersen, Environmental Science
- Laura Moore, Geology
- Richard Salter, Computer Science



- Robert Marcus, Computer Science



- Richard Gass, Physics
- Karen Tomko, Computer Science and Engineering



- Art Ross, Physics

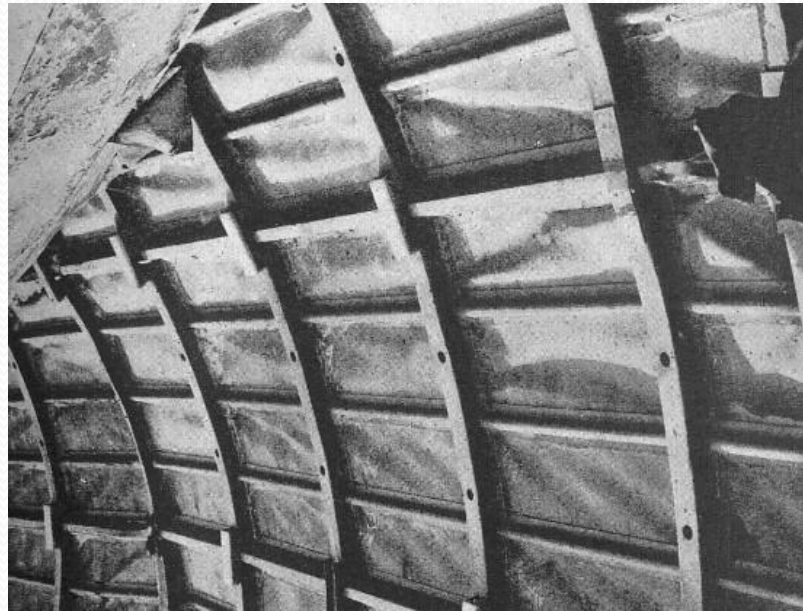
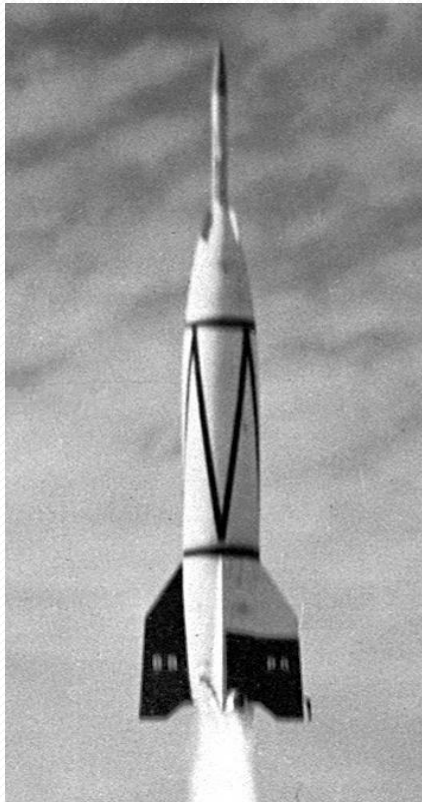


- Ron Taylor, Computer Science

An Upper Level Undergraduate Computational Science Module: “Analysis of Damaged Panels in Supersonic Flow”

- Integrates Communications, Mathematics, Mechanical Engineering, and Computer Science: Model with Plug-In Components.
- Good place to show computational matrix methods applied to an “old problem”.

Show Relevance: Engineering and Mathematical History





JSTOR

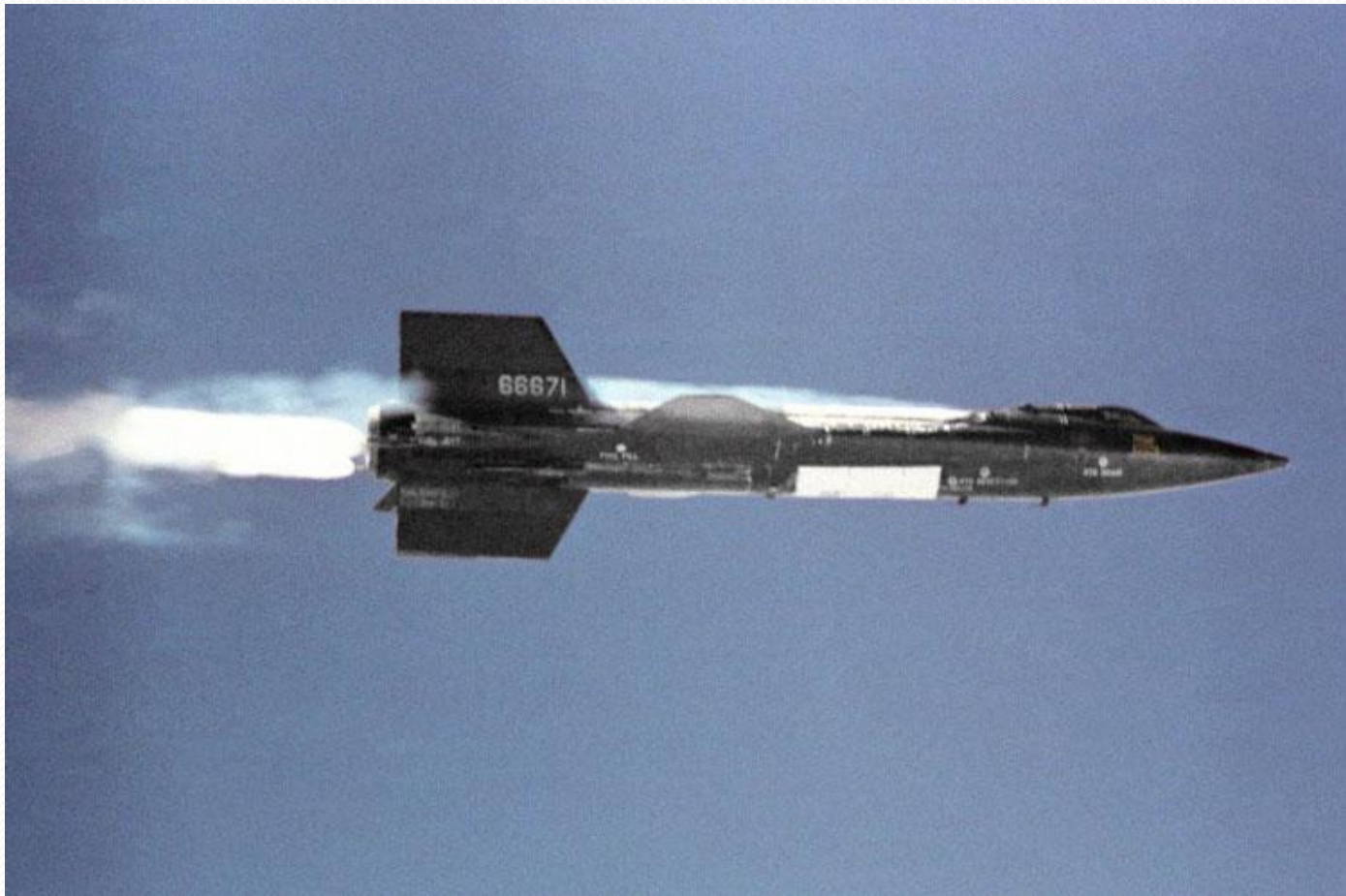
**“How I Became a Torchbearer for Matrix Theory”,
Olga Taussky-Todd,
The American Mathematical Monthly, Nov 1988**

(A) *The Geršgorin theorem*

The war was on, WWII, and I was working in London at the National Physical Laboratory under R. A. Frazer in the flutter group. I was assigned to the study of flutter in supersonic aircraft, which leads to boundary value problems in hyperbolic partial differential equations. Hence this work did not immediately contribute to my matrix enthusiasm. However, I had read Frazer’s article on how the flutter calculations were to be carried out. A large group of young girls, drafted into war work, did the calculation on hand-operated machines, following the instructions of Frazer and his assistants.

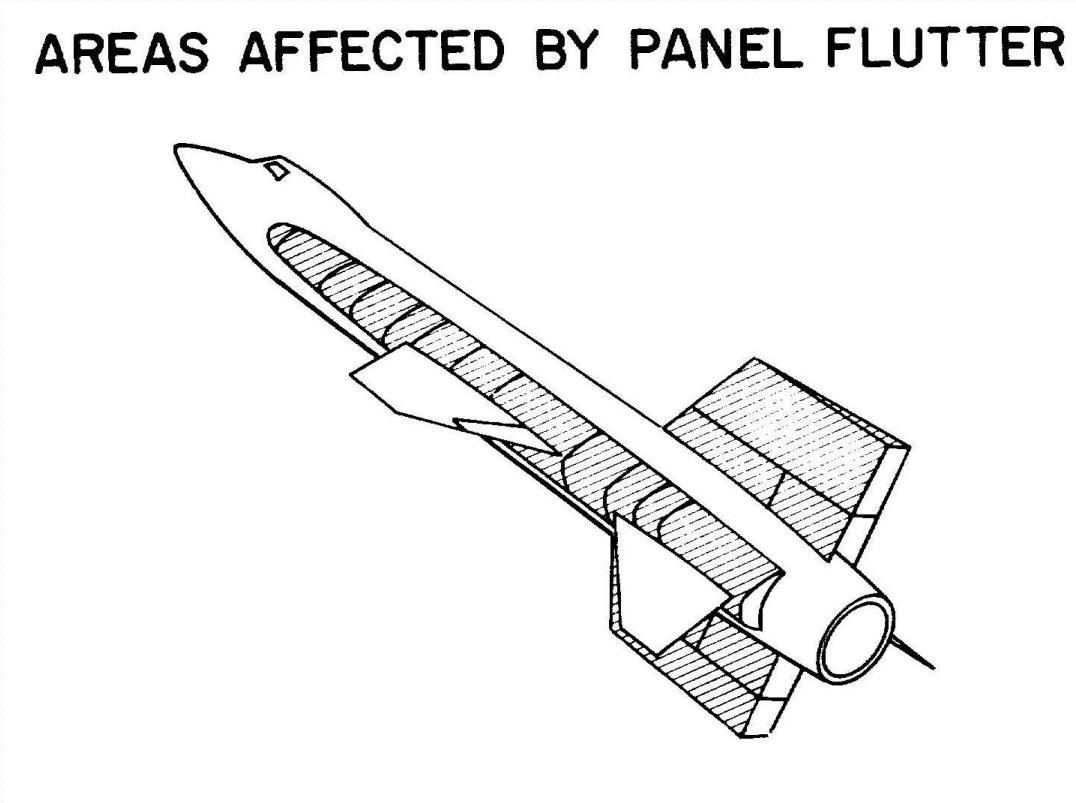
In the case that I was working on, the question came down to showing that a certain 6×6 matrix of the form $-\omega^2 A + i\omega B + C$, where ω , the flutter parameter, is taken as 1 in the example (see Fig. 1), had no real eigenvalue to the left of the small circles. The matrix entries were in the neighborhood of 20 or so, but in itself that told us nothing about the whereabouts of the eigenvalues.

History: 1950s-60s



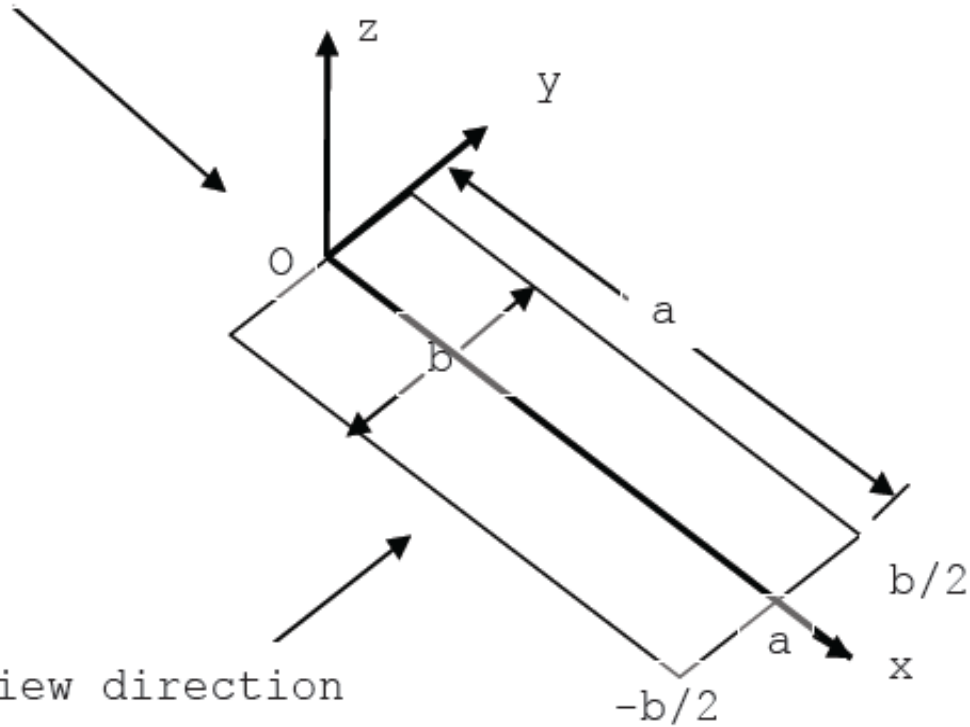
X-15 “Rocket Plane”

AREAS AFFECTED BY PANEL FLUTTER



Model: Historical Approach

V_∞ , air speed;
freestream flow direction



Governing Equations: What Does It Mean Term by Term?

The governing partial differential equation (PDE)

$$D(x) \frac{\partial^4 w}{\partial x^4} + m(x) \frac{\partial^4 w}{\partial t^2} = f(x, t) \quad , 0 < x < a$$

where

$$f(x, t) = f_F(x, t) + f_P(x, t) + f_V(x, t)$$

Simple supports for boundary conditions and initial displacement and velocity distribution. Reduce to matrix problem via Galerkin.

Galerkin Method: MATLAB Symbolic Math Toolbox Very Useful – Changing Assumed Functions is Easy

$$K_{Dij} = \int_0^a \sin\left(\frac{i\pi x}{a}\right) \frac{d^4}{dx^4} \left(\sin\left(\frac{j\pi x}{a}\right)\right) dx$$

The MATLAB Symbolic Mathematics Toolbox is very useful for computing the resulting terms. For example the terms involving $D(x)\frac{\partial^4 w}{\partial x^4}$ are computed using the code shown in the table below.

Line Number	MATLAB Statement
1	<code>% File: trygalerkin01.m</code>
2	<code>clear ; close all ; clc ; format compact ;</code>
3	<code>n=5;</code>
4	<code>syms x;</code>
5	
6	<code>% Define the shape functions:</code>
7	
8	<code>for i = 1:n,</code>
9	<code> phi(i) = sin(i*pi*x);</code>
10	<code>end</code>
11	
12	<code>% Set up the KD matrix due to plate bending stiffness:</code>
13	
14	<code>for i=1:n,</code>
15	<code> for j=1:n,</code>
16	<code> KD(i,j) = int(phi(i)*diff(phi(j),x,4),x,0,1);</code>
17	<code> end</code>
18	<code>end</code>

Interesting Eigenvalue Problem

$$(K + \mu P + \lambda A - \nu^2 M)c = 0$$

$K =$ Plate Stiffness /
Foundation Matrix

$\mu =$ Edge load
parameter

$P =$ Geometric Stiffness
Matrix

$\lambda =$ Dynamic Pressure
parameter

$A =$ Air load (skew-
symmetric)
matrix

$\nu =$ Frequency
parameter

$M =$ Mass matrix

Design Optimization Problem: Analytic Rates of Change of Eigenvalues

$$(K + \mu P + \lambda A - \nu^2 M)c = 0$$

- K or M can be functions of design (e.g. coefficients in a panel thickness distribution).
- Implicit differentiation to get rates of change of critical edge loading, air loading or frequency.

Interaction Diagram: 1 of 2

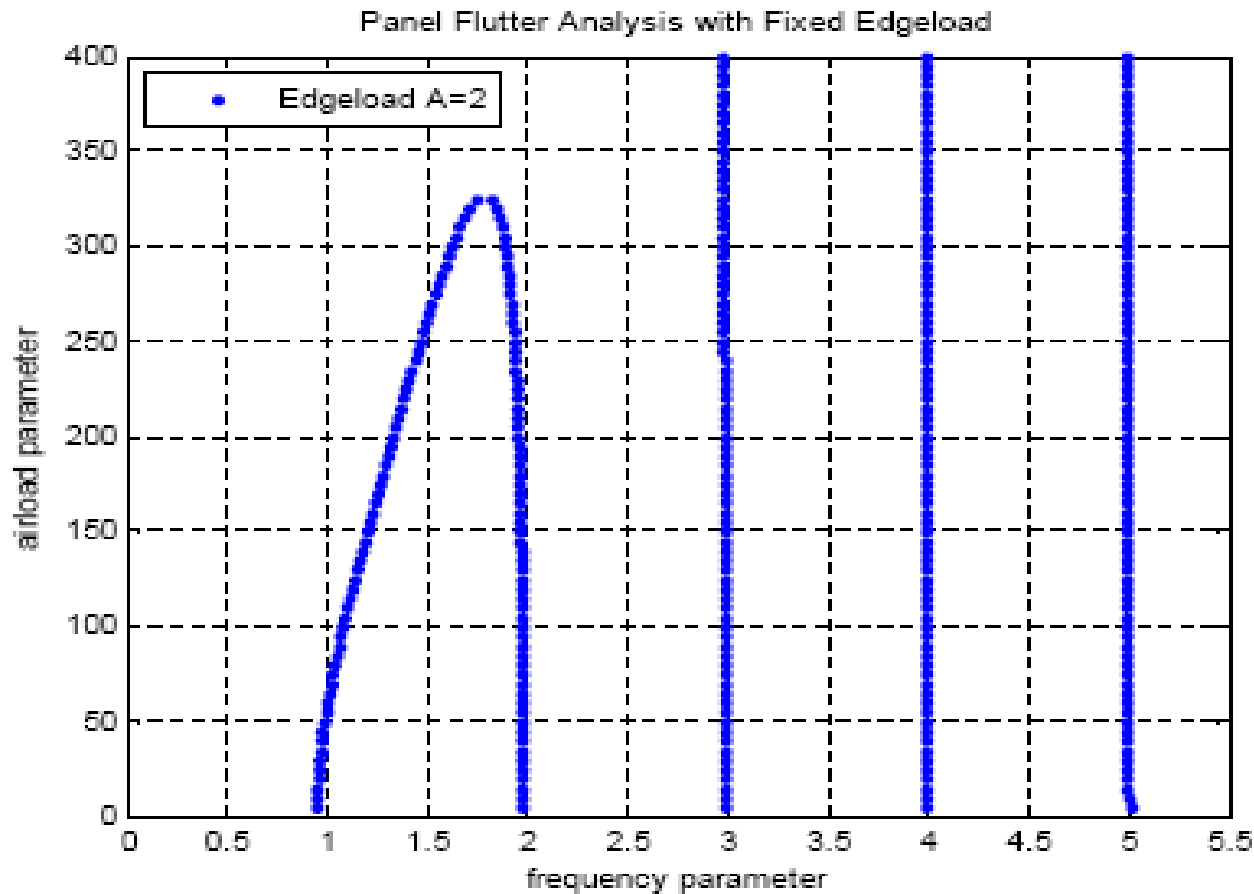


Figure 4.1. Flutter Instability at Edgeloading of 2.

Interaction Diagram: 2 of 2

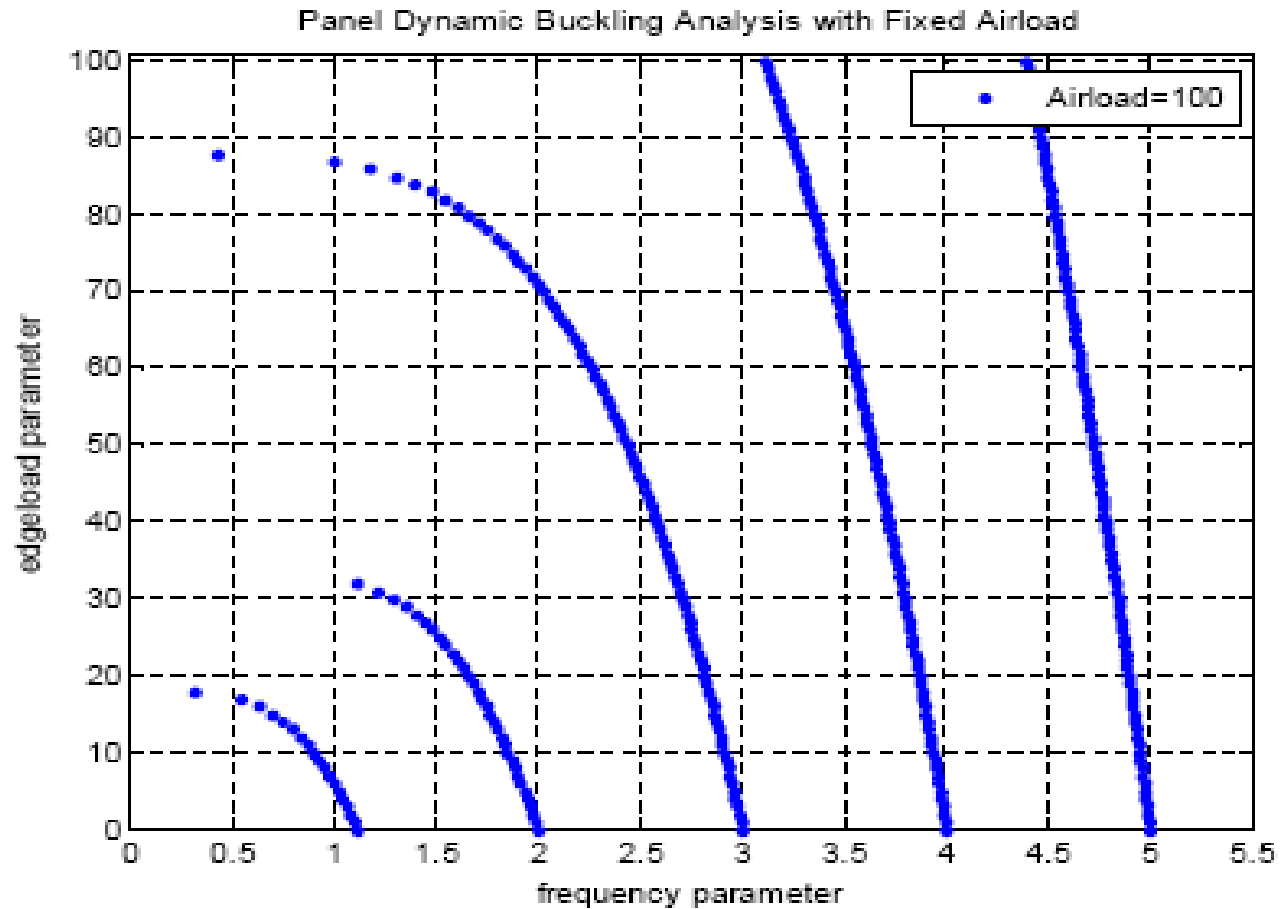


Figure 4.2. Buckling Instability at Airload of 100.

Ideas for “More Matrix Modules”

- Google Search Engine Project
 - Large-Scale Eigenvalue Problems
- Matrix computations applied to “Sudoku matrices”: K
 - Interesting numerical properties, e.g. $\rho(K)=45$
 - Pose: How many matrix concepts/computations are relevant to the K matrix?
 - Pose: Can you write code to create a
 - ‘*Rotated K* ’ or a
 - ‘*Mirror Image of K* ?’”

Key Benefit of Modules: Helping to Develop Professional Work Standards

- How do I know if my computer simulations are “correct?” (No more answers in back of book!)
- What does it mean to have “correct” results?
- Do my results “make sense?”
- Are the physical units correct and consistent?
- Have I used several approaches/ methods/software packages and am I getting results with understandable variance?
- What are the ethical and social implications of my work?

Summary

- Computational Science demands strong background in computation and matrix algebra.
- Modules are useful especially for well prepared undergraduates at the Junior and Senior level.
- Help to develop communication skills and professional work standards.
- Students need to understand mathematical rigor at least not confuse a numerical result with a proof!

Department of Computer Science and Engineering

