On nonautonomous linear systems of differential and difference equations with R-symmetric coefficient matrices

William F. Trench

Trinity University, San Antonio, Texas 78212-7200, USA
Mailing address: 659 Hopkinton Road, Hopkinton, NH 03229 USA
email: wtrench@trinity.edu

Abstract

Let $\mathbb{C}^{n\times n}(\mathcal{J})$ denote the set of continuous $n \times n$ matrices on an interval $\mathcal{J}$. We say that $R \in \mathbb{C}^{n\times n}(\mathcal{J})$ is a nontrivial k-involution if $R = P\left(\bigoplus_{\ell=0}^{k-1} \zeta^\ell I_{d_\ell}\right) P^{-1}$ where $\zeta = e^{-2\pi i/k}$, $d_0 + d_1 + \cdots + d_{k-1} = n$, and $P' = P\left(\bigoplus_{\ell=0}^{k-1} U_\ell\right)$ with U_ℓ and $U_\ell^{-1} \in \mathbb{C}^{d_\ell \times d_\ell}(\mathcal{J})$. We say that $A \in \mathbb{C}^{n\times n}(\mathcal{J})$ is R-symmetric if $R(t)A(t)R(t)^{-1} = A(t)$, $t \in \mathcal{J}$, and we show that if A is R-symmetric then solving $x' = A(t)x$ or $x' = A(t)x + f(t)$ reduces to solving k independent $d_\ell \times d_\ell$ systems, $0 \le \ell \le k-1$. We consider the asymptotic behavior of the solutions in the case where $\mathcal{J} = [t_0, \infty)$. Finally, we sketch analogous results for linear systems of difference equations.