

Department: MATHEMATICAL SCIENCES

Semester Hours: 3

Course Title and Number: MATH 430 - ADVANCED CALCULUS I

Course Description: Re-examination of the calculus of functions of one variable: convergence, continuity, differentiation, the mean-value theorem, and the Riemann integral.

PRQ: MATH 232 and MATH 240, or MATH 334.

Course Objectives:

- To reason rigorously in mathematical arguments.
- To communicate mathematics clearly and efficiently.
- To think creatively in abstract situations.
- To understand the unifying nature of abstract mathematical treatments.
- To understand fundamental concepts of the real number system, functions and the calculus.

Content:

- Real Numbers: Peano axioms; elementary operations on sets; rational numbers and their arithmetic; bounded sets and completeness of the real numbers; absolute value; triangle inequality.
- Functions, Sequences, and Limits: Mappings, functions, and sequences; limit of a sequence; limit theorems for sequences; limit of a function at a point; sequential criteria for limit; limit theorems for functions; monotone Sequences.
- Continuity: Topology of the real line; Bolzano-Weierstress theorem; Heine-Borel theorem; continuity and uniform continuity; operations with continuous functions; properties of continuous functions (Intermediate Value Theorem, etc.).
- Differentiation: The derivative; sequential criteria for the existence of the derivative; chain rule; Mean Value Theorem; Cauchy's mean value theorem; L'Hospital's rule; Taylor's theorem and Taylor's formula with remainder; Extreme values.
- Integration: Basic facts about Riemann Integral; example of an integrable function on  $[0, 1]$  which is discontinuous at rationals; properties of the definite integral; Fundamental Theorems of Calculus; further properties of Integrals.

Course requirements: Class time is devoted to an extremely careful dialogue with students, the theoretical foundations of the calculus. Homework assignments are collected on a regular basis and intensively analyzed. Construction of proofs and knowledge of real numbers. Reading mathematics. Discussion of levels of understanding of concepts. Hour examination and a comprehensive final examination.

Assessment Instruments: Judgment on student involvement and depth of contributions in class discussions. Homework proof writing and problem solving. Hour examinations. Final examination.

Current textbook: Edward D. Gaughan, **Introduction to Analysis**, 5th Edition, American Mathematical Society (2009).