Department: MATHEMATICAL SCIENCES
Course Title and Number: MATH 444 - Linear Programming & Network Flows

Course Description: This course deals with optimization problems that have additional structure such as linearity, quadratic objective, or maybe only convexity. The computational process known as the simplex algorithm will be developed in great detail and it is shown how it in principle is capable of solving all linear problems with a finite number of constraints. The idea of a 'dual' problem is in this case beautifully illustrated by the data tracked in the simplex tableau. Duality connects with the so-called Lagrange multipliers seen in general constrained optimization. The concept of convexity permeates the whole subject and points at wide-ranging generalizations. Applications of linear problems are seen in the context of networks and the so-called transportation problem. More immediate problems involve things like: minimizing cost subject to constraints, maximizing profit in many business settings, and especially so-called Game Theory. The impact of uncertainty in the data of the problem is seen to be of considerable interest and the discipline of 'sensitivity analysis' will be discussed in some depth. The issue of computational speed in worst case scenarios versus generic or 'average' cases is also significant and it is addressed by so-called interior-point algorithms that penetrate the feasible set rather than jumping from vertex to vertex. Some of the complexities encountered are visible in two-dimensional problems and this will be the launching point for the whole course. Parallel to the development of mathematical ideas software tools will be introduced and actual implementations will be part of the course. Key techniques will involve linear algebra but when the material involves non-linear objectives and constraints, then there is an immediate need for a more extensive apparatus.

Credits: 3
Prerequisites & Notes
PRQ: PRQ: MATH 239 or MATH 240, or consent of department.

Course Objectives:

- To reason rigorously in mathematical arguments.
- To communicate mathematics clearly and efficiently.
- To appreciate the unifying nature of abstract mathematical treatments.
- To develop a firm foundation in optimization techniques.
- To experience, hands-on, the issues involved when producing software implementations.

Content:

- Linear optimization in two dimensions using graphs.
- Linear algebra and row operations.
- Matrix notation and matrix algebra.
- The simplex algorithm.
- Phase-1 and phase-2.
- The dual problem.
- The fundamental theorem of linear programming.
- Complementary slackness.
- Sensitivity theory.
- Game theory using graphs.
- Game theory via the simplex algorithm.
- Quadratic objectives.
- Convexity theory.

- Convex optimization.
- Extensions to general non-linear problems.
- Computer implementations and related issues.

Course Requirements: Attendance is crucial as material is presented from a variety of sources. There will be two midterm exams. The first midterm will involve calculations related to the simplex algorithm. The second midterm concerns duality, sensitivity analysis, and game theory. The comprehensive final exam involves all of the above as well as a little bit of general non-linear optimization.

Assessment Instruments: Assignments throughout the semester, midterm examinations and the final exam.

Useful References: *Linear and Nonlinear Programming, 2nd ed.* by David G. Luenberger, Addison Wesley (1989), *Introduction to Mathematical Programming,* by Russell C. Walker, Prentice Hall (1999), *The Compleat Strategyst,* by J. D. Williams Dover (1986)

GRADING: Your grade will be based on a total of 500 points as follows:
Attendance and Assignments - 100 points
Midterm exams - 200 points
Final exam - 200 points

GRADING SCALE:

A: 425
A-: 400
B+: 375
B: 350
B-: 325
C+: 300
C: 275
D: 250

FINAL EXAM: The final exam is on Monday, December 7th, 4:00--5:50 p.m.

ACADEMIC CONDUCT: Academic honesty and mutual respect (student with student and instructor with student) are expected in this course. Mutual respect means being on time for class and not leaving early, being prepared to give full attention to class work, not reading newspapers or other material in class, not using cell phones or pagers during class time, and not looking at another student's work during exams. Academic misconduct, as defined by the Student Judicial Code, will not be treated lightly.

DRC STATEMENT: Northern Illinois University is committed to providing an accessible educational environment in collaboration with the Disability Resource Center (DRC). Any student requiring an academic accommodation due to a disability should let his or her faculty member know as soon as possible. Students who need academic accommodations based on the impact of a disability will be encouraged to contact the DRC if they have not done so already. The DRC is located on the 4th floor of the Health Services Building, and can be reached at 815-753-1303 (V) or drc@niu.edu.