Department: MATHEMATICAL SCIENCES Semester Hours: 3

Course Title and Number: MATH 480 - Number Theory

Course Description: Divisibility, primes, congruences, quadratic reciprocity, Diophantine equations, continued fractions, and selected topics.

PRQ: MATH 420 or consent of department.

Course Objectives.

- To reason rigorously in mathematical arguments.
- To communicate mathematics clearly and efficiently.
- To more fully understand the structure of the set of integers and the deeper properties of the prime numbers.
- To see patterns and undelying theorems based on examples and computation.
- To appreciate what we know, i.e., can prove, about the integers (Prime Number Theorem, Dirichlet's Theorem, Fermat's Last Theorem).
- To appreciate what we don't know, i.e., cannot prove (Twin Prime Conjecture, Goldbach Conjecture, Number of primes represented by a polynomial of degree 2 or higher).

Content:

- Divisibility: Division algorithm; greatest common divisor; least common multiple; Euclidean algorithm; primes; fundamental theorem of arithmetic.
- Congruences: Linear congruences; Euler ϕ-function; theorems of Fermat, Euler and Wilson; perfect numbers, Chinese remainder theorem.
- Primitive roots: Order of an integer modulo n; characterization of those moduli having primitive roots.
- Quadratic reciprocity: Quadratic residues; Legendre symbol; law of quadratic reciprocity and supplements.
- Arithmetical Functions: The functions σ, τ, ϕ and μ; Möbius inversion; recurrence relations.
- Diophantive equations: Sums of two and four squares; Pythagoren triples; Pell's equation.
- Continued Fractions; Farey sequences; Contiguous Fractions.

Course Requirements: Discussion of new tools and strategies in problem solving. View problems from multiple viewpoints and depth in the study of number theory. Solution of more advanced problems and proofs. Regular homework. In-class presentations. Exams.

Assessment Instruments: Judgment on student involvement and depth of contributions in class discussions and presentations at the board. Homework problem solving and proof construction. Midterm. Final exam.

Current textbook: Instructor Notes.

References/textbooks:
Introduction to Number Theory by Niven, Zuckermann, and Montgomery
Elementary Number Theory and its Applications by Rosen
Introduction to Analytic Number Theory by Apostol