

MATH 211
EXAM 1
SPRING 2005

Professor K. Datta

Name _____
Section _____

1. (15 pts.) Let

$$f(x) = \begin{cases} 2x + 1 & \text{for } x \leq -1 \\ x - 2 & \text{for } x > -1. \end{cases}$$

(a) $\lim_{x \rightarrow -1^+} f(x) =$

(b) $\lim_{x \rightarrow -1^-} f(x) =$

(c) Is $f(x)$ continuous at $x = -1$? Give reasons.

2. (15 pts.) Evaluate the following limits:

(a) $\lim_{x \rightarrow 1} \left(\frac{1}{x^3} + 3x^2 - 5\sqrt{x} + 2 \right) =$

(b) $\lim_{x \rightarrow 2} \frac{2x^2 - 3x - 2}{x^2 + x - 6} =$

(c) $\lim_{x \rightarrow -2} \frac{(x+2)^2}{(x^2-4)} =$

3. (20 pts.) Find derivative of the following functions:

(a) $f(x) = \frac{1}{2}x^2 - \frac{3}{x} + 2x + 5$

(b) $f(x) = 2\sqrt{x} + 5x^{10} + \frac{3}{x^2} + 1$

(c) $f(x) = \frac{2x^3 + 3x^2}{2x + 3}$ (Use Quotient rule)

(d) $f(x) = \left(5x^2 + \frac{2}{x^2}\right)(3x + x^3)$ (Use Product rule)

4. **(15 pts.)** Find the equation of the tangent line to the graph $y = \frac{x+1}{x-1}$ at the point $(2, 3)$.

5. (20 pts.)

(a) Using the limit definition of the derivative, find $f'(x)$ if $f(x) = \sqrt{x+1}$.

(b) Find the slope of the tangent line of $f(x) = \sqrt{x+1}$ at $x = 3$

6. (15 pts.) The profit (in dollars) from the sale of x car seats for infants is given by

$$P(x) = 4x - .025x^2 - 5000 \quad 0 \leq x \leq 2400.$$

(a) Find the average change in profit if production is changed from 800 to 850 car seats.

(b) Find the instantaneous rate of change of profit at a production level of 800 car seats and give interpretation of the result.