1. (9 points) Carefully state the following theorems, making sure you have the hypotheses correct.

- The Extreme Value Theorem

- The Mean Value Theorem

- Fermat’s Theorem

2. (6 points) From which of the three theorems above can it be argued that if a drain pipe fills a 10 gallon bucket in 3 minutes, then at some point during that period the drain was flowing at a rate of over 150 gallons per hour. Explain.

3. (15 points)

   (a) If Newton’s method, with initial value $x_1 = 4$, is used to approximate $\sqrt{8}$, the positive zero of $f(x) = x^2 - 8$, what are $x_2$ and $x_3$?

   (b) Sketch the graph of $f(x) = x^2 - 8$. Show on your graph how Newton’s Method constructs $x_2$ from $x_1$. 
4. (10 points) Show that the equation \(3 - 5x^3 - 6x^5 = 0\) has exactly one solution. State any theorems you use.

5. (15 points) Find the following limits. Justify your conclusions.

(a) \(\lim_{x \to \infty} \frac{x^{3/2} - 2x^2 + 1}{3x^2 - 5x^3}\)

(b) \(\lim_{x \to \infty} (3x + 1) \sin \left( \frac{1}{x} \right)\)

(c) \(\lim_{x \to \infty} (x - \sqrt{x^2 + 3x + 4})\)
6. (10 points) Let \( f(x) = 11x + \frac{22}{x} - 10. \)

(a) Name the theorem that guarantees that \( f \) has both an absolute maximum and an absolute minimum on the interval \([-4, -1]\).

(b) Find the absolute maximum and absolute minimum of \( f \) on \([-4, -1]\).

7. (15 points) Use calculus to find the point on the curve \( y = \sqrt{x} \) that is closest to the point \((0, 108)\).
8. (20 points) Let \( f(x) = \frac{x}{x^3 - 1} \).

(a) Find any vertical and horizontal asymptotes of the graph of \( f \).

(b) Find the intervals of increase and decrease of \( f \) and all points \((a, f(x))\) for which \( f(a) \) is a local maximum or a local minimum.

(c) Find the intervals on which \( f \) is concave upward and those on which it is concave downward. Find all inflection points \((b, f(b))\) of \( f \).

(d) Sketch a good graph of \( f \) that plots all intercepts, local extrema, inflection points and vertical and horizontal asymptotes, and is consistent with all your answers above.