Math 232 Final (Fall 2010)   Section (   )

NAME (Print): ______________________________

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1. (20 pts) Let P$(1, 2, 3)$ and Q$(1, -1, -2)$ be two points in the three dimensional space.

(a) Find the area of the triangle with vertices at P, Q, and the origin.

(b) Find an equation of the line through P$(1, 2, 3)$ and that is perpendicular (orthogonal) to the plane through P, Q, and the origin.

2. (10 pts) Find an equation of the tangent plane to the surface $z = e^{xy}$ at the point $\mathrm{P}(1, 1, e)$.

3. (10 pts) Suppose the equation $xy + e^{xyz} - z - e^{y} = 0$ implicitly defines each of the variables as functions of the other two variables. Use implicit partial derivative to find $\dfrac{\partial z}{\partial x}$ at the point $\mathrm{P}(1, 1, 1)$.

4. (10 pts) Decide a scalar q so that two planes $qx + y + z = 3$ and $x - y + 5z = 0$ are perpendicular.

5.(16 pts) Let $f(x, y) = x^2 - 5xy$.

a) Find the directional derivative of the function at (1,2) in the direction of $\vec{v} = 3\vec{i} + 4\vec{j}$.

b) Find the direction in which the function increases most rapidly at (1,2). Then find the derivative in that direction.

6. (14 pts) Find the local maxima, local minima, and saddle points of

$f(x, y) = \frac{1}{3}x^3 + \frac{1}{3}y^3 - xy + 4,$ if there is any.

7. (15 pts) Use the method of Lagrange multipliers to find the point on the plane $x+2y+3z=6$ that is closest to the origin. (Hint: minimize the square of the distance: $d^2 = x^2 + y^2 + z^2$ to make algebra simpler.)

8. (14 pts) Evaluate $\int_0^1 \int_y^1 \frac{e^x}{x}\, dx\, dy$ (Hint: reverse the order of integration.)

9. (14 pts) Compute the double integral $\int\int_D (y - 5x)^3(2y + x)^4\, dA$ where D is the region bounded by $y = 5x$, $y = 5x + 1$, $2y = -x - 1$, and $2y = -x + 1$.

(Hint: Use the transformation $u = y - 5x$ and $v = 2y + x$.

10. (20 pts) Set up the following triple integrals to find the volume of the upper hemi-sphere of radius a, $x^2 + y^2 + z^2 = a^2$ and $z \geq 0$ (Do not compute)

a) a triple integral in cylindrical coordinates :

b) a triple integral in spherical coordinates :

11. (32 pts) Let $\mathbf{F} =< xy^2 + 2y,\ x^2y + 2x + 2 >$ be a vector field in the two dimensional space.

a) Show that $\mathbf{F}$ is conservative.

b) Find a potential function f for the field $\mathbf{F}$.

c) Compute the line integral $\int_c \mathbf{F} \cdot d\mathbf{r}$ where c is a smooth curve

$\mathbf{r}(t) =< e^t,\ 1 + t >,\ 0 \le t \le 1.$

d) Compute the line integral $\int_c \mathbf{F} \cdot d\mathbf{r}$ where c is a <u>closed</u> curve

$\mathbf{r}(t) =< 2\sin t,\ \cos t >,\ 0 \le t \le 2\pi.$

12.(10 pts) A wire of density $\delta(x,y) = x$ lies along the curve

$\mathbf{r}(t) = < t,\, t^2 >,\ 0 \le t \le 2$. Find the mass of the wire.

13.(15 pts) Evaluate $\int_c -y^3\,dx + x^3\,dy$ where c is a circle given by

$\mathbf{r}(t) = < 2\cos t,\, 2\sin t >,\ 0 \le t \le 2\pi$. (Hint: Use Green's theorem)