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Math 240 Final Exam, Dec 11, 2002

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There are 200 possible points. Calculators are not allowed.

1.(20 points) Let

$$A = \begin{bmatrix} 1 & 3 & 0 & 0 & 1 \\ 2 & 6 & 0 & 1 & 2 \\ -1 & -3 & 1 & 1 & 1 \\ 2 & 6 & 1 & 2 & 4 \end{bmatrix}$$

a. Find a basis for the column space of A .

b. Find a basis for the nullspace of A .

2. (20 points) Is the set of vectors $\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix}, \begin{bmatrix} 4 \\ -1 \\ 3 \end{bmatrix} \right\}$ linearly independent?

3. (15 points) Let S be the ordered basis $\left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \end{bmatrix} \right\}$ for \mathbb{R}^2 . Let $v = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$. Find $[v]_S$, the coordinate vector for v with respect to S .

4. (15 points) Let W be the subspace of R^4 spanned by $\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 3 \\ 1 \\ 4 \\ 0 \end{bmatrix}$.
Find an orthogonal basis for W .

5.(20 points) Let V be the vector space of functions with basis $S = \{1, x, e^x\}$. Let W be the vector space of functions with basis $T = \{1, e^x\}$. Let $L : V \rightarrow W$ be the linear transformation given by $L(f(x)) = f'(x)$ for all $f(x)$ in V . Find the matrix representation of L with respect to S and T .

6. (15 points) For each of the following matrices, find its determinant. (Hint: Look for a quick way to do these.)

a.

$$\begin{bmatrix} 1 & 2 & 31 & 11 \\ 2 & 4 & 17 & 23 \\ 3 & 6 & 7 & 5 \\ 4 & 8 & 19 & 29 \end{bmatrix}$$

b.

$$\begin{bmatrix} 0 & 2 & 31 & 11 & 13 \\ 3 & 0 & 17 & 23 & 53 \\ 0 & 0 & 1 & 5 & 6 \\ 0 & 0 & 0 & 1 & 13 \\ 0 & 0 & 0 & 0 & 2 \end{bmatrix}$$

7.(20 points) a. Let V be the vector space of all functions from R to R . Let W be the set of all functions f in V such that $f(0) = 2$. Is W a subspace of V ? Explain.

b. Let V be the vector space of all 2×2 matrices. Let W be set of all matrices A in V such that $A^T + A = 0$. Is W a subspace of V ? Explain.

8.(20 points) Let

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & -4 \\ 0 & -2 & -1 \end{bmatrix}.$$

If possible, find an invertible 3×3 matrix P and a diagonal matrix D such that $P^{-1}AP = D$.

9.(10 points) Let A be a 5×5 matrix. Let $L : R^5 \rightarrow R^5$ be the linear transformation such that $L(v) = Av$ for all v in R^5 . Assume that A has rank 3. Answer the following questions. Always give a brief reason for your answer.

a. Is the matrix A invertible?

b. Is the range of L all of R^5 ?

c. What is the dimension of the kernel of L ?

d. Are there vectors v and w in R^5 such that $v \neq w$ but $L(v) = L(w)$?

e. Is 0 an eigenvalue for A ?

10. (15 points) Let V and W be vector spaces, and let $L : V \rightarrow W$ be a linear transformation. Let v_1, v_2 , and v_3 be vectors in V . Prove that if $\{L(v_1), L(v_2), L(v_3)\}$ is linearly independent, then $\{v_1, v_2, v_3\}$ is linearly independent.

11. (15 points) Let V be a vector space with an inner product. Prove that for all vectors v and w in V ,

$$\|v + w\|^2 + \|v - w\|^2 = 2\|v\|^2 + 2\|w\|^2.$$

12. (15 points) Let A be a $n \times n$ matrix such that $A^2 = A$. Prove that if λ is an eigenvalue for A , then $\lambda = 1$ or $\lambda = 0$.