Dimensional Analysis and a Model Ship Experiment

This project is an example of how dimensional analysis may be used to guide an experiment on a small scale model of a large system in order to obtain information about the large system. It also points out that this method does have limitations.

The variables involved in studying a propeller which is pushing a ship through water include: \( D \), the diameter of the propeller; \( V \), the speed of the ship; \( \rho \), the density of the water; \( F \), the thrust force of the propeller; \( n \), the rotational speed (in revolutions/second) of the propeller; \( g \), the acceleration due to gravity; and \( \mu \), the viscosity of water. It is clear that the geometry of the ship’s hull is also involved. Since this geometry and \( V \) are closely related, and since we may want to control \( V \) in other ways, we will use only \( V \) in our model. Gravity appears in our model to allow for the surface wave effects of the propeller.

**Problem 1.** Find the dimensions of the variables \( D, V, \rho, F, n, g, \) and \( \mu \).

**Problem 2.** Use dimensional analysis to derive the equation

\[
F = \rho D^2 V^2 \phi\left(\frac{nD}{V}, \frac{gD}{V^2}, \frac{\mu}{\rho D V}\right),
\]

where \( \phi \) is some (unknown) function.

**Problem 3.** If the effects of viscosity are assumed to be negligible, explain briefly why we may write

\[
F = \rho D^2 V^2 \Theta\left(\frac{nD}{V}, \frac{gD}{V^2}\right),
\]

for some (unknown) function \( \Theta \).

A ship with a propeller 20 feet in diameter is designed to move in water at a speed of 25 feet per second when the propeller turns at 2 revolutions per second. Before constructing the ship, we’d like to know what the thrust force of the propeller will be under these conditions. To accomplish this, we build a 1/10 scale model of the ship to be tested in a model ship basin, in which we can measure the thrust force of the propeller on the model ship, and we can vary the speed of the model ship. We assume that we may neglect viscosity.

There is a certain speed, \( V' \), and rotational speed, \( n' \), at which the model ship and its propeller should be run, so that when we measure the thrust of the propeller on the model ship, we can calculate the thrust force of the large propeller.
Problem 4. Find $V'$ and $n'$ and describe a method for calculating the thrust force, $F$, of the large propeller.

(HINT: Since we know $D, V,$ and $n$ for the large ship, we could compute $F$ from (2), if we knew $\Theta\left(\frac{nD}{V}, \frac{gD}{V^2}\right)$. But $F'$, the measured thrust force of the model, also satisfies (2), with $D', V'$, and $n'$ replacing $D, V$, and $n$. Thus, what is needed is a means of finding $\Theta\left(\frac{nD}{V'}, \frac{gD}{V'^2}\right)$ by using this information.)

Problem 5. Using $V'$ and $n'$ from Problem 4, compute the ratio $\frac{F'}{F}$.

We turn now to our assumption that viscosity is negligible. The factor in equation (1) involving viscosity, $\frac{\mu}{\rho DV}$, is the reciprocal of the Reynolds number of the system, $\frac{\rho DV}{\mu}$.

Problem 6. Using the values of $D$ and $V$ given for the large ship, and $D'$ (the propeller diameter of the model) and $V'$ as computed for the model, compute the ratio of the Reynolds numbers for the two systems, $\frac{\text{model}}{\text{full scale}}$. Explain why running the model, using $V'$ and $n'$, will not necessarily predict accurately the thrust force of the large propeller if viscosity is not negligible.