

MATH 430
FINAL
Fall 2006

Prof. Harris

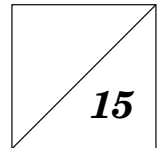
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1. Prove that for any $C \in \mathbb{R}$, the equation

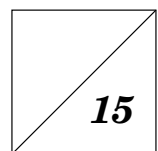
$$x^3 + 3x + C = 0$$

has at most one solution in the interval $[-1, 1]$.

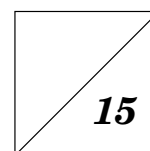


2. Use L'Hôpital's rule to evaluate

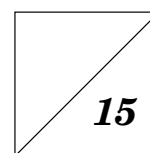
$$\lim_{x \rightarrow 0} \frac{x^2 \sin x}{\sin x - x \cos x}$$



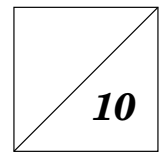
3. Use the definition of the derivative to find the derivative of $f(x) = 2\sqrt{x} + 1$ at $x_0 \in (0, \infty)$. Does $f'(0)$ exist? Explain.



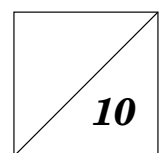
4. Prove, using ε and δ that $f(x) = x^2$ is continuous at any $x \in [10, 21)$. Is it uniformly continuous on this interval? State any theorems that you use.



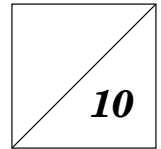
5. Prove that the set $S = \left\{ \left(\frac{x}{2}, n^2 \right) \mid x \in \mathbb{Q}, n \in \mathbb{J} \right\}$ is countable.



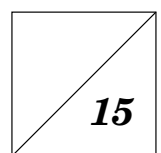
6. Let S be a non-empty subset of \mathbb{R} which is bounded below. Let $x = \inf\{S\}$. If $x \notin S$, prove that x is an accumulation point of S .



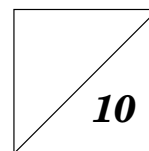
7. The set $(0, 1]$ is not compact. Find an infinite open cover with no finite subcover.



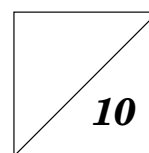
8. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be differentiable. If $f'(x) > 0$ for all $x \in \mathbb{R}$, prove that $F(x) = f(x) + 8$ is increasing. What, if any, extra conditions would need to be imposed for $G(x) = (f(x))^2$ to be increasing?



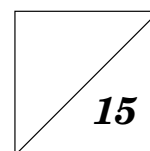
9. If $c > 1$ prove that $\{\sqrt[n]{c}\}$ is convergent. What is it convergent to?



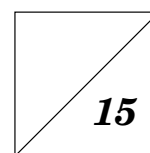
10. Prove directly that $f(x) = x$ is Riemann integrable on $[0, 1]$.



11. If $a_n, b_n > 0$ for all $n \in J$, $\{a_n^2\}$ is convergent and $\{b_n^2\}$ is Cauchy, prove that $\{a_n b_n\}$ is convergent. State any theorems that you use.

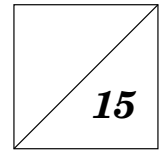


12. Must every bounded sequence in \mathbb{R} be convergent? Must every bounded sequence contain a convergent subsequence? Is every convergent sequence bounded? Give examples.

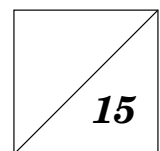


13. Show that

$$2^{2^x} = 6x \text{ for some } x \in (0, 1)$$



14. Let $f : [a, b] \rightarrow \mathbb{R}$ be continuous on $[a, b]$, differentiable on (a, b) with $f(a) = f(b) = 0$. Prove that for each $k \in \mathbb{R}$ there is a $c \in (a, b)$ with $f'(c) = kf(c)$. Hence, or otherwise, show that it is always possible to solve the equation $\cos x = k \sin x$ for $x \in (0, \pi)$ for all $k \in \mathbb{R}$



15. $f : [1, 2] \rightarrow [1, 2]$ is continuous with $f([1, 2]) = [1, 2]$. Prove that there exists $c \in (1, 2)$ with $f(c) = c$

