Department of Mathematical Sciences

QUALIFYING EXAMINATION FOR THE PH.D. IN MATHEMATICAL SCIENCES
COMPREHENSIVE EXAMINATION FOR THE M.S. IN MATHEMATICAL SCIENCES

These written examinations are given three times each year, just before the start of each semester (mid-January, mid-June, and mid-August). The following individual exams are available: algebra, analysis (real and complex), differential equations, numerical analysis, mathematics education, statistics: probability and inference, and statistics: linear models and Bayesian statistics. With the exception of the mathematics education exams noted below, all exams are three hours in length for the Qualifying Examination and two hours for the Comprehensive Examination. Syllabi for each of these individual exams are below.

Qualifying Examination

The following guidelines apply to doctoral students and also to master's students who are considering the possibility of going on for a doctorate. The Qualifying Examination consists of two individual exams from above, chosen as follows depending on the student's area of focus.

Mathematics, Core Course Group A: algebra and analysis.
Mathematics, Core Course Group B: mathematics education and one more of the student's choosing.
Mathematics, Core Course Group C: statistics: probability and inference, and statistics: linear models and Bayesian statistics.
Mathematics, Core Course Group D: differential equations and numerical analysis.

Successful completion of the Qualifying Examination is passing both of these individual written exams. It is typically expected that the student will attempt the first exam within two years of entering the program. If the Graduate Studies Committee determines that the student has failed an individual exam, then the student will be given the chance to retake the failed exam once more. If the Graduate Studies Committee determines that the student has failed an individual exam for the second time, the committee will recommend either the student complete a master's degree and leave the program, or simply leave the program.

Comprehensive Examination

The following guidelines apply to students pursuing a terminal master's degree. The Comprehensive Examination consists of the two-hour version of two individual exams from above (with the exception of the mathematics education exam described below with the syllabi), chosen as follows depending on the student's degree specialization. The guidelines for passing/failing here are similar to those above for the Qualifying Examination.

Pure Mathematics: algebra and analysis.
Applied Mathematics: analysis and differential equations.
Computational Mathematics: differential equations and numerical analysis.
Mathematics Education: mathematics education and one more of the student's choosing.

Syllabus for Algebra

Groups: groups, subgroups, normal subgroups, homomorphism theorems, Sylow theorems, structure theorem for finite abelian groups, Jordan-Hölder theorem, solvable groups.

Fields and Galois theory: characteristics, prime fields, algebraic and transcendental extensions, separability, perfect fields, normality, splitting fields, Galois group, fundamental theorem of Galois theory, solvability by radicals, structure of finite fields.

Linear algebra: linear independence, basis, dimension, direct sums, linear transformations and their matrix representations, linear functions, dual spaces, determinants, rank, eigenvalue and eigenvector, minimal and characteristics polynomials, canonical forms.

Rings and Modules: rings, homomorphisms, integral domains, Euclidean domains, principal ideal domains, unique factorization domains, field of fractions, polynomial rings, maximal, prime and primary ideals, chain conditions, Noetherian rings, Artinian rings, Wedderburn-Artin theorem, Jacobson radical, modules, simplicity, semisimplicity, finitely generated modules over a principal ideal domain.

Primary References:

Jacobson, Basic Algebra I, II
Hungerford, Algebra
Isaacs, Algebra: A Graduate Course
Rotman, The Theory of Groups
Garling, A Course in Galois Theory
Dummit and Foote, Abstract Algebra
Curtis, Linear Algebra: An Introductory Approach
Hoffman and Kunze, Linear Algebra

Secondary References:

Beachy and Blair, Abstract Algebra with a Concrete Introduction
Fraleigh, A First Course in Abstract Algebra
Herstein, Topics in Algebra
Anton, Elementary Linear Algebra

A student with basic background knowledge in linear algebra who has taken MATH 620 and MATH 621 will have been exposed to the necessary material to take the three-hour version (qualifying exam) of this exam; linear algebra plus MATH 620 should be adequate for the two-hour version.

Syllabus for Analysis

Real Analysis: Lebesgue measure and the Lebesgue integral on the line, sigma algebras, Borel sets, outer measure, measurable sets and functions, Egoroff's theorem, Lusin's theorem, Fatou's lemma, monotone convergence theorem, Lebesgue convergence theorem, differentiation of monotone functions and of an integral, absolute continuity.

Complex Analysis: construction and properties of complex numbers, topology of the complex plane, complex differentiation, the Cauchy-Riemann equations, analytic functions, power series, elementary functions and their properties, complex integration and Cauchy's theorem, applications of Cauchy's theorem, Liouville's theorem, Schwarz's lemma and the maximum modulus theorem, residue theory and applications.

References: Ahlfors, Complex Analysis
Aliprantis and Burkinshaw, Principles of Real Analysis
Aliprantis and Burkinshaw, Problems in Real Analysis
Apostol, Mathematical Analysis
Conway, Functions of One Complex Variable
Royden, Real Analysis
Needham, Visual Complex Analysis

A student who has taken MATH 630 and MATH 632 will have been exposed to the necessary material to take the three-hour version (qualifying exam) of this exam; MATH 630 and MATH 540 should be adequate for the two-hour version.

Syllabus for Differential Equations

Ordinary Differential Equations: existence and uniqueness of solutions of systems of linear and nonlinear ordinary differential equations, successive approximations, explicit solutions of constant coefficient linear systems including matrix exponentials, variation of parameters method for solving non-homogeneous systems, stability, oscillation, continuous dependence of solutions.

Partial Differential Equations: linear, quasi-linear, and nonlinear first-order equations; the classical mathematical models for the vibrating string, heat conduction and gravitational potential; characteristics, classification and canonical forms for second-order equations; the Cauchy problem and the Cauchy-Kovalevsky theorem; Holmgren's uniqueness theorem; basic results for elliptic, parabolic and hyperbolic linear equations in one and several space dimensions; systems of linear and quasi-linear equations.

References: Coddington and Levinson, Theory of Ordinary Differential Equations
Jordan and Smith, Nonlinear Ordinary Differential Equations
Waltman, A Second Course in Ordinary Differential Equations
Evans, Partial Differential Equations
John, Partial Differential Equations
Weinberger, A First Course in Partial Differential Equations

A student who has taken MATH 636 and MATH 642 will have been exposed to the necessary material to take the three-hour version (qualifying exam) of this exam; MATH 636 and MATH 542 should be adequate for the two-hour version.

Syllabus for Numerical Analysis

Numerical Analysis: floating-point arithmetic, roundoff errors, problem conditioning, algorithm stability. Polynomial interpolation: Lagrange and Newton forms, divided differences, error formula. Solution of nonlinear equations: bisection, Newton's method, fixed-point iteration, rates of convergence. Orthogonal polynomials and least-squares approximation. Numerical integration: basic and composite rules, Gaussian quadrature, adaptive quadrature, Romberg integration. Numerical solution of ordinary differential equations: Euler's method, Runge-Kutta methods, multistep methods, absolute stability and stiffness.

Numerical Linear Algebra: basic algorithms and norms. Gaussian elimination, LU factorization, positive-definite matrices, Cholesky factorization. Sensitivity, condition number, backward error analysis. Orthogonal matrices, Householder and Givens transformations, QR decomposition, the least-squares problem. The singular value decomposition. Eigenvalues and eigenvectors: the power method and extensions, similarity transforms and the Schur decomposition, reduction to Hessenberg and tridiagonal forms, the QR Algorithm (a.k.a. Francis's Algorithm).

References: Ascher and Greif, A First Course in Numerical Methods
Burden and Faires, Numerical Analysis
Golub and Van Loan, Matrix Computations
Kincaid and Cheney, Numerical Analysis: Mathematics of Scientific Computing
Trefethen and Bau, Numerical Linear Algebra
Watkins, Fundamentals of Matrix Computations

A student who has taken MATH 664 and MATH 662 will have been exposed to the necessary material to take the three-hour version (qualifying exam) of this exam; MATH 534 and MATH 535 should be adequate for the two-hour version.

Syllabus for Mathematics Education

At the Comprehensive Examination level, this is a written three hour exam covering MATH 610 and at least two courses selected from among MATH 611-617, depending on the courses taken by the student. Any student desiring to take this exam should consult with the Director of Graduate Studies to determine the exact nature of this exam.

At the Qualifying Examination level this is a two part written examination: three hours in the morning and three hours in the afternoon (with a break in between). This is an examination on the learning and teaching of mathematics, and on related policy documents and recommendations. Research-based literacy on students' and teachers' understandings of specific mathematical concepts is expected. Literacy with specific original research published in theses, dissertations, books, and research-based periodicals is expected. Further, the student is expected to link her/his knowledge about students' mathematical thinking and knowledge-building with how classroom instruction and assessment may be guided to enhance the meaningful learning of mathematics. Emphasis will be placed on in-depth analysis, synthesis and the ability to interpret and critique existing publications.

Students are expected to have knowledge of the development of mathematics education as an academic field of study including the impact of the work of learning theorists, leading mathematics education researchers, reform efforts, and the related work of professional organizations such as NCTM, AMTE, AERA, NCSM, and MAA. In addition, the student should have knowledge of the results of effective professional development models in mathematics education and the related research.

See the Director of Graduate Studies for an up-to-date list of suggested references and reading.

Students who have taken MATH 610 and at least two other graduate-level mathematics education courses at the 600-level will have been exposed to the necessary material to take the Comprehensive Examination. Students taking the Qualifying Examination should have taken at least three graduate-level mathematics education courses at the 600-level in addition to MATH 610.

Syllabus for Statistics: Probability and Inference

Probability: probability spaces, measures, measurable functions and algebra of events, random variables, expectations, characteristic and moment generating function, discrete, continuous, mixed and multivariate probability distributions, sequences of random variables and various modes of convergence, Borel-Cantelli

Lemma and 0-1 laws, weak and strong law of large numbers, convergence in distributions and central limit theorems, conditional expectations and conditions distributions. Additional topics vary depending on the coverage in STAT 670 and may include martingales, Brownian motion and other stochastic processes, infinitely divisible and stable distributions, asymptotics, and various probability inequalities.

Inference: exponential families, location and scale families, hierarchical models and mixture distributions, sampling distributions, properties of sample mean and variance from normal distribution, sufficiency principle, complete families, point estimation including unbiasedness, maximum likelihood and Bayesian estimation, consistency, hypothesis testing and interval estimation. Additional topics vary depending on the coverage in STAT 672 and may include statistical decision theory, asymptotics, and higher-order theory.

References: Gut, Probability: A Graduate Course
Pollard, A User's Guide to Measure Theoretic Probability
Shorack, Probability for Statisticians
Casella and Berger, Statistical Inference
Mukhopadhyay, Probability and Statistical Inference
Schervish, Theory of Statistics
Young and Smith, Essentials of Statistical Inference

A student who has taken STAT 670 and STAT 672 will have been exposed to the necessary material to take this exam.

Statistics: Linear Models and Bayesian Statistics

Linear Models: multivariate normal distribution, distribution of quadratic forms, linear models and design matrix of less than full rank, estimation and distribution theory, generalized least squares, hypothesis testing and distribution theory for F-test, confidence interval and regions, multiple comparisons, analysis of variance. Additional topics vary depending on the coverage in STAT 673 and may include polynomial regression, departure from assumptions and diagnostics, prediction, and model selection.

Bayesian Statistics: Bayesian inference, loss function and risk, one parameter models and posterior inference, conjugate priors, non-informative priors, multi-parameter models, Bayesian computation, Gibbs sampling, Markov chain Monte Carlo methods and applications in different areas. Additional topics may include decision theory, theoretical and convergence properties of Markov chain samplers, Bayesian model checking, model selection and assessment criteria, hierarchical models and Bayesian survival analysis.

References: Christensen, Plane answers to complex questions (3rd ed.)
Seber and Lee, Linear Regression Analysis (2nd ed.)
Ravishankar and Dey, A First Course in Linear Model Theory
Rencher and Schaalje, Linear Models in Statistics (2nd ed.)
Robert, The Bayesian Choice From Decision-Theoretic Foundations to Computational Implementation (2nd ed.)
Christensen, Johnson, Branscum and Hanson, Bayesian Ideas and Data Analysis: An Introduction for Scientists and Statisticians
Carlin and Louis, Bayesian Methods for Data Analysis (3rd ed.)
Berger, Statistical Decision Theory: Foundations, Concepts, and Methods
Gelman, Carlin, Stern and Rubin, Bayesian Data Analysis (2nd ed.)

A student who has taken STAT 673 and STAT 680 will have been exposed to the necessary material to take this exam.