Section K | MATH 211 Homework | Due Friday, 8/30/96

Professor J. Beachy | | Average: 15.1 / 20

18, page 18: If $f(x) = \dfrac{x^2}{x^2-1}$, find $f(\frac{1}{2})$, $f(-\frac{1}{2})$, and $f(a+1)$.

22, page 18: When a solution of acetylcholine is introduced into the heart muscle of a frog, it diminishes the force with which the muscle contracts. The data from experiments of A.J.Clark are closely approximated by a function of the form

$$R(x) = \frac{100x}{b+x}, \qquad x \geq 0,$$

where x is the concentration of acetylcholine (in appropriate units), b is a positive constant that depends on the particular from, and $R(x)$ is the response of the muscle to the acetylcholine, expressed as a percentage of the maximum possible effect of the drug.

(a) Suppose that $b = 20$. Find the response of the muscle when $x = 60$.

(b) Determine the value of b if $R(50) = 60$ — that is, if a concentration of $x = 50$ units produces a 60% response.

53, page 20: Compute $f(1)$, $f(2)$, and $f(3)$ for $f(x) = \begin{cases} \sqrt{x} & \text{for } 0 \leq x < 2 \\ 1+x & \text{for } 2 \leq x \leq 5 \end{cases}$.

11, page 29: Determine the intercepts of the graph of the function $f(x) = -\frac{1}{4}x + 3$.

32, page 31: Sketch the graph of the function $f(x) = \begin{cases} 4x & \text{for } 0 \leq x < 1 \\ 8-4x & \text{for } 1 \leq x < 2 \\ 2x-4 & \text{for } 2 \leq \end{cases}$.

7, page 36: Express $f(x) + g(x)$ as a rational function, for $f(x) = \dfrac{2}{x-3}$, $\qquad g(x) = \dfrac{1}{x+2}$.

17, page 36: Express $\dfrac{f(x)}{g(x)}$ as a rational function, for $f(x) = \dfrac{x}{x-2}$, $\qquad g(x) = \dfrac{5-x}{5+x}$.

31, page 36: If $f(x) = x^2$, find $f(x+h) - f(x)$ and simplify.

10, page 45: Use the quadratic formula to solve the equation $x^2 - \sqrt{2}x - \frac{5}{4} = 0$.

33, page 46: Solve the equation $\dfrac{21}{x} - x = 4$.

MATH 211 QUIZ 1 8/30/96

Prof. J. Beachy Avg. 13.4 / 20 Show all necessary work.

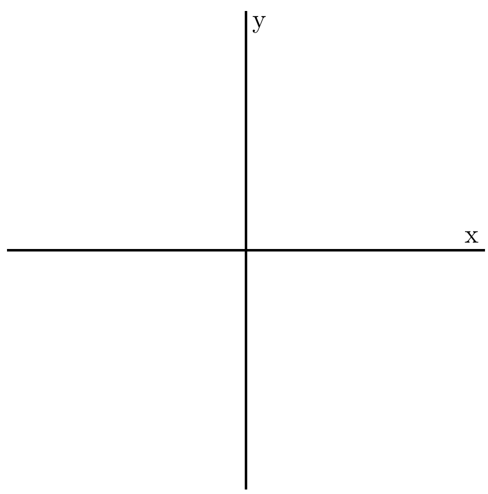
1. (5 points) For $f(x) = \dfrac{x}{x-2}$ and $g(x) = \dfrac{5-x}{5+x}$,
express $f(x+2) + g(x+2)$ as a rational function, and simplify.

2. (5 points) If $f(x) = x^2$, find $\dfrac{f(x+h) - f(x)}{h}$ and simplify.

3. (10 points) Find the indicated values of the function, and then sketch the graph of the function on the axes given below.

$$f(x) = \begin{cases} 2x & \text{for } -3 \le x \le -1 \\ -2 & \text{for } -1 < x < +1 \\ x-1 & \text{for } +1 \le x \le +3 \end{cases}$$

$f(-3) =$ $f(-2) =$ $f(-1) =$ $f(0) =$ $f(1) =$ $f(2) =$ $f(3) =$



Section K MATH 211 Homework NAME ______ Avg. 16.9 / 20 ______

Professor J. Beachy Due Friday, 9/6/96 Circle recitation time: T 1:00 Th 1:00 Th 2:00

69, page 53: Simplify: $\dfrac{(-27x^5)^{2/3}}{\sqrt[3]{x}} =$

18, page 64: Assume that a rectangular box with a square base has no top. Let x be the width of the base, and let h be the height of the box. The material needed to construct the base costs \$5 per square foot, and the material needed to construct the sides costs \$4 per square foot. Write an equation expressing the fact that the total cost of material is \$150.

24, page 64: A cellular telephone company estimates that if it has x thousand subscribers, then its monthly profit is $P(x)$ thousand dollars, where $P(x) = 12x - 200$.

(a) How many subscribers are needed for a monthly profit of 160 thousand dollars?

(b) How many new subscribers would be needed to raise the monthly profit from 160 to 166 thousand dollars?

18, page 81: Find the equation of the line that has the points $(5, -3)$ and $(-1, 3)$ on the line.

#52, page 84: Temperatures of $32°F$ and $212°F$ correspond to temperatures of $0°C$ and $100°C$. Suppose the the linear equation $y = mx + b$ converts Fahrenheit temperatures to Celsius temperatures. Find m and b. What is the Celsius equivalent of $98.6°F$?

MATH 211	QUIZ 2	NAME	Avg. 17.1 / 20
Prof. J. Beachy	9/6/96	Circle recitation time:	T 1:00 Th 1:00 Th 2:00

1. (7 points)
(a) State the quadratic formula, which gives the solution(s) to the equation $ax^2 + bx + c = 0$.

(b) Use the quadratic formula to solve the equation $x^2 - \sqrt{2}x - \frac{5}{4} = 0$.

2. (6 points) (Exercise 23, page 64)
A frozen yogurt stand makes a profit of $P(x) = .40x - 80$ dollars when selling x scoops of yogurt per day.
(a) Find the break-even sales level, that is, the level at which $P(x) = 0$.

(b) What sales level generates a daily profit of \$30?

3. (7 points) (Exercise 23, page 81)
(a) Find the slope of the line given by the equation $5x + 2y = -4$.

(b) Find the equation of the line that goes through the point (0,17) and is parallel to the line whose equation is $5x + 2y = -4$.

MATH 211 QUIZ 3 SOLUTIONS NAME ______ Avg. 11.8 / 20

Prof. J. Beachy 9/13/96 Circle recitation time: T 1:00 Th 1:00 Th 2:00

1. (6 points) For all parts of this question, use $f(x) = \sqrt{x}$.
   (a) Use the power rule to find the derivative $f'(x)$.

$f(x) = x^{\frac{1}{2}}$ $\qquad$ $f'(x) = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2x^{\frac{1}{2}}} = \frac{1}{2\sqrt{x}}$

(b) Find $f'(4)$.

$f'(4) = \frac{1}{2\sqrt{4}} = \frac{1}{4}$

(c) Use the information from parts (a) and (b) to find the equation of the tangent line to the curve $y = \sqrt{x}$ at the point $(4, 2)$.

The tangent line has a slope of $\frac{1}{4}$ and goes through the point $(4, 2)$. It has the following equation.

$y = \frac{1}{4}(x - 4) + 2$

2. (6 points) Find the following limit.

$$\lim_{x\to 1} \frac{x^2 + x - 2}{x^2 - 1} = \lim_{x\to 1} \frac{(x-1)(x+2)}{(x-1)(x+1)} = \lim_{x\to 1} \frac{x+2}{x+1} = \frac{1+2}{1+1} = \frac{3}{2}$$

3. (8 points)
   (a) State the limit definition of the derivative of a function $f(x)$.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

(b) Use the limit definition of the derivative of a function to find $f'(2)$, where $f(x) = 3x^2 - x$.

First, we can compute $f(2 + h) = 3(2 + h)^2 - (2 + h)$ and $f(2) = 3(2)^2 - 2 = 12 - 2 = 10$.

$$f'(2) = \lim_{h\to 0} \frac{f(2+h) - f(2)}{h} = \lim_{h\to 0} \frac{[3(2+h)^2 - (2+h)] - 10}{h} = \lim_{h\to 0} \frac{3(4 + 4h + h^2) - 2 - h - 10}{h}$$

$$= \lim_{h\to 0} \frac{12 + 12h + 3h^2 - 12 - h}{h} = \lim_{h\to 0} \frac{11h + 3h^2}{h} = \lim_{h\to 0} \frac{(11 + 3h)h}{h} = \lim_{h\to 0} 11 + 3h = 11$$

The first hour test is scheduled for this Friday, September 20. The problems will be like quiz and homework questions–it will not be machine graded, and I will give partial credit.

The test will cover sections 0.1–0.6 and 1.1–1.7. The emphasis will be on Chapter 1, on the derivative. Be sure to study sections 1.3, 1.4, 1.6, and 1.7 very carefully.

You will be asked for a valid photo ID at the test. Bring your NIU ID card or driver's license.

MATH 211 TEST I NAME Avg. 72.7 / 100 Std. dev 17.5

Prof. J. Beachy 9/20/96 Circle recitation time: T 1:00 Th 1:00 Th 2:00

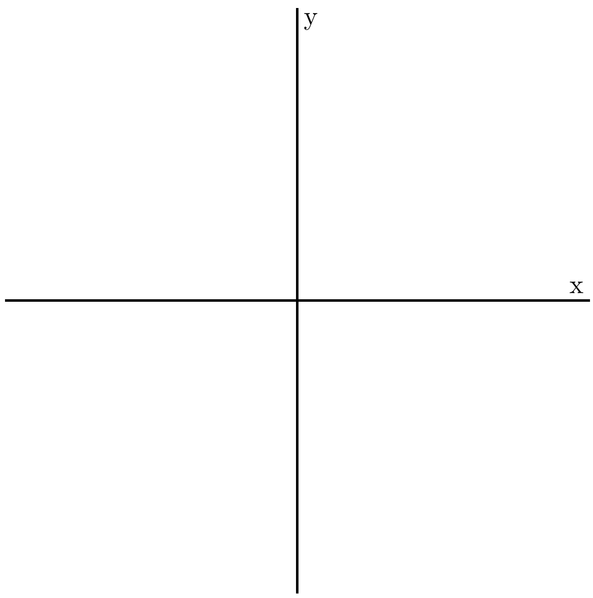
NO CALCULATORS! Be sure to show all necessary work. In your answers, only do the obvious arithmetic simplifications.

1. (8 points) Use the quadratic formula to solve the equation $16x^2 - 24x + 9 = 0$. Check your answer by substituting it into the equation.

2. (7 points) Find the composite function $h(g(x))$, where $h(x) = -x^2 + 1$ and $g(x) = \dfrac{x}{1-x}$. *Simplify your answer.*

3. (10 points) Sketch the graph of the function on the axes given below.

$$f(x) = \begin{cases} 2 & \text{for } -5 \leq x < -2 \\ |x| & \text{for } -2 \leq x \leq +2 \\ -x + 4 & \text{for } +2 < x \leq +5 \end{cases}$$



4. (25 points) Find the derivative $f'(x)$ of each of the following functions. Use the formulas we have learned–do *not* use the limit definition of the derivative.

(a) $f(x) = 5x^4 - 7x^3 + 6x - 3$

(b) $f(x) = \dfrac{1}{x^2} - \dfrac{1}{x}$

(c) $f(x) = \sqrt{1 - x^2}$

(d) $f(x) = \dfrac{1}{x^3 + 4x}$

(e) $f(x) = (x^4 - 2x + 7)^{4/3}$

5. (7 pts) Find the equation of the line tangent to the curve $y = (x^2 - 15)^4$ at $x = 4$.

6. (6 pts) If $A = 2\pi hr + 2\pi r^2$, find the first and second derivatives of A with respect to r.

$$\frac{dA}{dr} =$$

$$\frac{d^2A}{dr^2} =$$

7. (12 pts) Find the following limits.

$$\lim_{x\to 2} \frac{x^2 + x - 6}{x^2 - 5x + 6} =$$

$$\lim_{x\to 2} \frac{x^2 + x - 6}{x^2 + 5x + 6} =$$

8. (25 points)

(a) State the limit definition of the derivative of a function $f(x)$.

$f'(x) =$

(b) Use the limit definition of the derivative of a function to find $f'(2)$, for the function $f(x) = x^2 - 3x$.

(c) Use the limit definition of the derivative of a function to find $f'(5)$, for the function $f(x) = \dfrac{1}{2x-3}$.

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MATH 211 QUIZ 4 NAME ______ Avg. 10.9 / 20

Prof. J. Beachy 9/27/96 Circle recitation time: T 1:00 Th 1:00 Th 2:00

1. (6 points) A toy rocket fired straight up into the air has height $s(t) = 160t - 16t^2$ feet after t seconds.

(a) What is the velocity after 2 seconds?

(b) What is the acceleration after 2 seconds?

(c) At what time will the rocket reach its highest point?

2. (14 points) In this question you will sketch the curve $y = x^3 - 6x^2 + 9x - 1$.

(a) Find the first derivative $y' =$

(b) Solve $y' = 0$ to find the points at which the tangent line is horizontal.

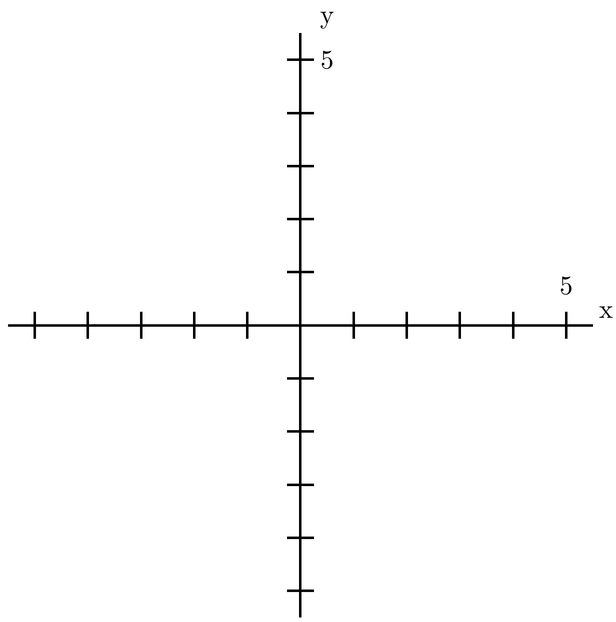
(c) Give the intervals (in terms of x) on which the curve is increasing and decreasing.

(d) Find the second derivative $y'' =$

(e) Solve $y'' = 0$ to find the point of inflection.

(f) Give the intervals (in terms of x) on which the curve is concave up or concave down.

(g) Using the above information, sketch the curve on the given axes. You should actually plot at least 5 points.



MATH 211	QUIZ 5	NAME ______ Avg. 10.9 / 20	
Prof. J. Beachy	10/4/96	Circle recitation time:	T 1:00 Th 1:00 Th 2:00

1. (4 points) Write down the objective equation and the constraint equation for the following problem (use P for the product):

Find two positive numbers, x and y, whose sum is 100 and whose product is as large as possible.

(a) The objective equation is:

(b) The constraint equation is:

2. (16 points) A carpenter has been asked to build an open box with a square base. The sides of the box will cost \$3 per square meter, and the base will cost \$ 4per square meter. What are the dimensions of the box of largest volume that can be constructed for \$48?

(a) (2 pts) What is to be maximized or minimized?

(b) (3 pts) Write down an equation that expresses the quantity in (a) as a function of one or more variables. This is the objective equation.

(c) (4 pts) Use the constraints given in the problem to simplify the objective equation to give a function of one variable.

(d) (5 pts) Differentiate, and use calculus to find the dimensions of the box of optimum size.
(e) (2 pts) Be sure to use either the first or second derivative to check that your answer gives a maximum.

Answer: Width = ______ Height = ______

MATH 211 HOMEWORK NAME ______________________

Prof. J. Beachy Due 10/9/96 Circle recitation time: T 1:00 Th 1:00 Th 2:00

This homework assignment is optional. If you choose to hand it it, your grade on this assignment will be averaged with your grade on QUIZ 5.

A cylindrical can is to hold 4π cubic inches (about 7 ounces) of frozen orange juice. The cost per square inch of constructing the metal top and bottom is twice the cost per square inch of constructing the cardboard side. What are the dimensions of the least expensive can?

(a) (2 pts) What is to be maximized or minimized? ______________________

(b) (5 pts) Write down an equation that expresses the quantity in part (a) as a function of one or more variables. This is the objective equation.

(c) (5 pts) Use the constraints given in the problem to simplify the objective equation to give a function of one variable.

(d) (5 pts) Differentiate, and use calculus to find the dimensions of the can of optimum size.

(e) (3 pts) Be sure to use either the first or second derivative to check that your answer gives a minimum.

Answer: Radius = __________ Height = __________

MATH 211 QUIZ 6 NAME ______ Avg. 12.5 / 20

Prof. J. Beachy 10/11/96 Circle recitation time: T 1:00 Th 1:00 Th 2:00

1. (10 points) A bookstore is attempting to determine the economic order quantity for a popular book. The book sells 8000 copies per year; it costs \$40 so process each order; the carrying cost is \$2 per book, figured on the maximum inventory. How many times a year should orders be placed?

If x is the number of orders placed per year, then the inventory cost is

$$C(x) = 40x + \frac{1600}{x} .$$

Find the value of x that minimizes the inventory cost.

2. (10 points) A company currently produces 2,000 units per week, and sells them at \$80 each. The production cost is a fixed \$10,000 per week, and \$50 per unit. The company would like to expand production, but estimates that to produce 4,000 units per week they would have to drop the price to \$60 per unit.

(a) Find the current profit.

(b) Assuming a linear demand curve, find the function which gives the profit P as a function of the number x of units produced per week.

MATH 211 TEST II NAME Avg. 65.5 / 100 Std. dev. 16.8

Prof. J. Beachy 10/18/96 Circle recitation time: T 1:00 Th 1:00 Th 2:00

NO CALCULATORS! Be sure to show all necessary work.

1. (30 points) Find the derivative of each of these functions.

(a) $f(x) = (x^4 + x^2)^{10}$

(b) $f(x) = (x+1)^2(x-3)^3$

(c) $f(x) = \dfrac{x^2 - 1}{x^2 + 1}$

(d) $f(x) = \dfrac{4x^2 + x}{\sqrt{x}}$

(e) $f(x) = \sqrt{\dfrac{x^2 - 1}{x^2 + 1}}$

2. (15 points) The graph of

$$f(x) = \frac{1}{3}x^3 - x^2 - 3x + 5$$

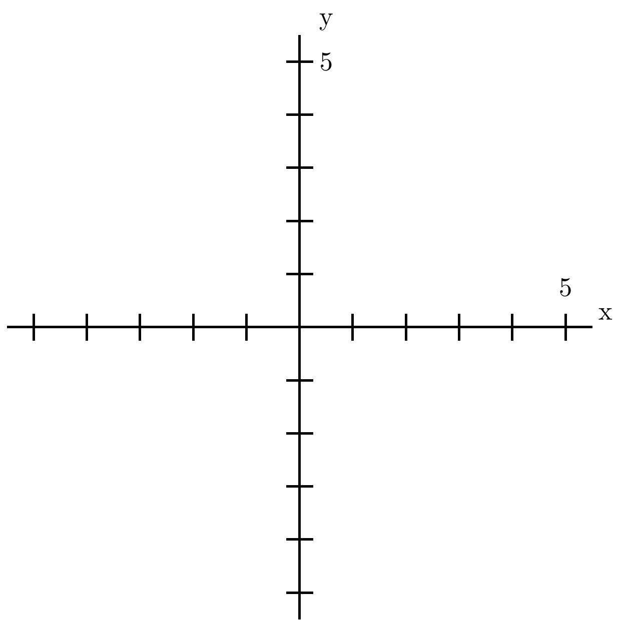
has one relative maximum point and one relative minimum point. Find the coordinates of these relative extreme points and use the second derivative to determine which point is the relative maximum and which point is the relative minimum. *You do not need to include a sketch of the graph of* $f(x)$.

3. (15 points) The demand equation for a monopolist is $p = 200 - 3x$ dollars, where p is the price per unit and x is the number of units produced. The cost function is $C(x) = 75 + 80x - x^2$ dollars for $0 \le x \le 40$. Determine the value of x and the corresponding price per unit that will maximize the total profit.

4. (20 points) On the axes below, sketch the graph of

$$y = \frac{1}{x} + \frac{x}{4}\ .$$

Find the intervals on which the graph is concave up or concave down.
On the graph, indicate all relative extreme points, and all inflection points.



5. (20 points) A closed rectangular box is to be constructed with a base that is twice as long as it is wide. Suppose that the total surface area must be 27 square feet. Find the dimensions of the box that will maximize the volume.

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GRADE	

MATH 211 QUIZ 7 NAME ______ Avg. 15.3 / 20

Prof. J. Beachy 10/25/96 Circle recitation time: T 1:00 Th 1:00 Th 2:00

1. Solve for x: $10^{1-x} = 100$

2. Solve for x: $(3^{2x} \cdot 3^2)^4 = 3$

3. Factor: $3^{10x} - 1 = (3^{5x} - 1)(\qquad\qquad)$

4. Solve for x: $e^{x^2-2x} = e^8$

5. Differentiate: $y = (1 + x^2)e^x$

6. Differentiate: $f(x) = \dfrac{e^x}{1 + e^x}$

MATH 211 | HOMEWORK | NAME ______ Avg. 12.9 / 20

Prof. J. Beachy | Due 11/1/96 | Circle recitation time: T 1:00 Th 1:00 Th 2:00

1. (4 pts) Solve the equation $\ln(\ln 2x) = 0$ for x.

2. (6 pts) For the function $f(x) = e^{-x} + 3x$, find the coordinates of each relative extreme point, and determine if the point is a relative maximum point or a relative minimum point.

3. (4 pts) If $f(x) = \ln \sqrt{x}$

then $f'(x) =$

4. (6 pts) Suppose that the total revenue function for a manufacturer is $R(x) = 300 \ln(x + 1)$, so that the sale of x units of a product brings in about $R(x)$ dollars. Suppose also that the total cost of producing x units is $C(x)$ dollars, where $C(x) = 2x$. Find the value of x at which the profit function $R(x) - C(x)$ will be maximized. Show that the profit function has a relative maximum and not a relative minimum point at this value of x.

MATH 211 QUIZ 8 NAME ______ Avg. 12.9 / 20

Prof. J. Beachy 11/8/96 Circle recitation time: T 1:00 Th 1:00 Th 2:00

1. (5 pts) Simplify: $e^{\ln x^2 + 3 \ln y}$

2. (5 pts) Simplify, then differentiate: $y = \ln \left[\sqrt{\frac{x^2 + x + 1}{x^3 - 1}} \right]$

3. (10 pts) Suppose that a certain bacterial culture grows at a rate proportional to its size. At time $t = 0$, approximately $20,000$ bacteria are present. In 5 hours there are $40,000$ bacteria.

(a) Determine a function that expresses the size of the culture as a function of time, measured in hours.

(b) How many bacteria are present in 10 hours?

MATH 211 TEST III NAME Avg. 66.2 / 100 Std. dev. 17.7

Prof. J. Beachy 11/15/96 Circle recitation time: T 1:00 Th 1:00 Th 2:00

NO CALCULATORS! Be sure to show all necessary work.

1. (25 points) Find the derivative of each of these functions.

(a) $f(x) = \ln |x^4 + x^2 + 1|$

(b) $f(x) = \dfrac{e^x - 1}{e^x + 1}$

(c) $f(x) = \left(e^{x^2} + x^2\right)^5$

(d) $f(x) = \ln\left(\dfrac{(x^4+1)^3 e^x}{\sqrt{x^2+1}}\right)$

(e) $f(x) = \ln(\ln(\sqrt{x}\,))$

2. (20 points)

(a) $\int (x^3 + 6x^2 - 1)\, dx =$

(b) $\int \left(\frac{2}{\sqrt{x}} - 3\sqrt{x} \right) dx =$

(c) Which of the following is equal to $\int \ln x \; dx$? Show your work, and circle the correct answer.

(i) $\frac{1}{x} + C$ (ii) $x \cdot \ln x - x + C$ (iii) $\frac{1}{2}(\ln x)^2 + C$

3. (10 points) Show, by differentiating, that the function $y = P_0 e^{kt}$ satisfies the equation

$$\frac{dy}{dx} = ky \ .$$

The terms P_0 and k are assumed to be constants.

4. (15 points) Radioactive cobalt 60 has a half-life of 5.3 years.

(a) Find the decay constant of cobalt 60.

(b) If the initial amount of cobalt 60 is 10 grams, show how to compute the amount present after 2 years and after 4 years.

5. (15 points) The world's population was 5.4 billion on January 1, 1991, and was expected to reach 6 billion on January 1, 1995. Assume that at any time the population grows at a rate proportional to the population at that time.

(a) Find the formula for $P(t)$, the world's population t years after January 1, 1991.

(b) Using this model of population growth, how many years does it take for the world's population to double in size?

6. (15 points) The function $f(x) = \dfrac{\ln x}{\sqrt{x}}$ has a relative extreme point for $x > 0$. Find the coordinates of the point. Is it a relative maximum, or a relative minimum point?

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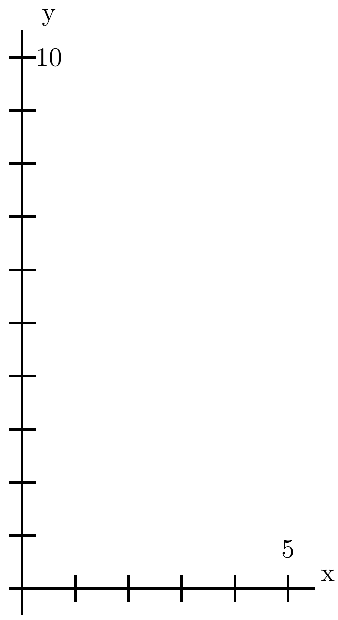
MATH 211 QUIZ 9 NAME Avg. 13.9 / 20

Prof. J. Beachy 11/22/96 Circle recitation time: T 1:00 Th 1:00 Th 2:00

1. (p.394 #5; 10 pts) Use a Riemann sum to approximate the area under the graph of $f(x) = x^2$ on the interval from $x = 1$ to $x = 3$. Use 4 subdivisions ($n = 4$), and evaluate the function at the left endpoint of each subinterval.



2. (p.409 #9; 5 pts) Evaluate this definite integral:

$$\int_1^4 3\sqrt{x}\, dx =$$

3. (p.410 #11; 5 pts) Evaluate this definite integral:

$$\int_0^5 e^{-2t}\, dt =$$

MATH 211	HOMEWORK	NAME ________ Avg. 18.9 / 20
Prof. J. Beachy	Due 12/6/96	Circle recitation time: T 1:00 Th 1:00 Th 2:00

This homework assignment is optional. In computing your quiz grade I will drop the lowest score. Your score on this homework (should you choose to accept the assignment) will replace the lowest of the remaining scores.

1. (5 pts) Let $u = x^3 + 1$.

$$\int 3x^2(x^3+1)^2 \, dx =$$

2. (5 pts) $\int 3x^2 e^{(x^3-1)} \, dx =$

3. (5 pts) $\int \frac{x}{\sqrt{x^2+1}} \, dx =$

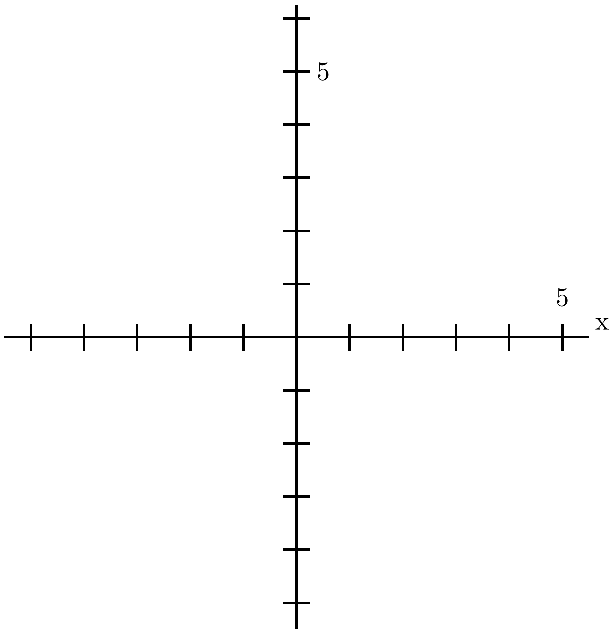
4. (5 pts) Let $u = \ln x$.

$$\int \frac{\ln \sqrt{x}}{x} \, dx =$$

MATH 211 QUIZ 10 NAME Avg. 11.9 / 20

Prof. J. Beachy 12/6/96 Circle recitation time: T 1:00 Th 1:00 Th 2:00

1. (p.421 #11; 10 pts) Find the area of the region bounded by the graphs of $y = x^2 + x$ and $y = 3 - x$. First find the points of intersection, and then graph the relevant parts of the two functions.



2. (5 pts) Find the average value of the function $f(x) = x^3$ over the interval from $x = 0$ to $x = 2$.

3. (5 pts) Use the formula $\int_a^b \pi[g(x)]^2\, dx$ to find the volume of the solid of revolution generated by revolving about the x-axis the region under the curve $y = \sqrt{r^2 - x^2}$ from $x = 0$ to $x = r$. (This is a hemisphere of radius r.)

MATH 211 FINAL EXAM NAME Avg. 67.8% Std. dev. 16.2

12/9/96 NO CALCULATORS! Show all necessary work. Be neat, clear, and brief.

1. (30 points) Find the derivative of each of these functions. You do not need to simplify your answers.

(a) $f(x) = 8x^3 - 7x + \dfrac{5}{x}$

(b) $f(x) = (x^2 + 3)^9$

(c) $f(x) = \dfrac{e^{-x} + 1}{e^x - 1}$

(d) $f(x) = e^{4x} + \ln x$

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2. (15 pts) Find the following limits.

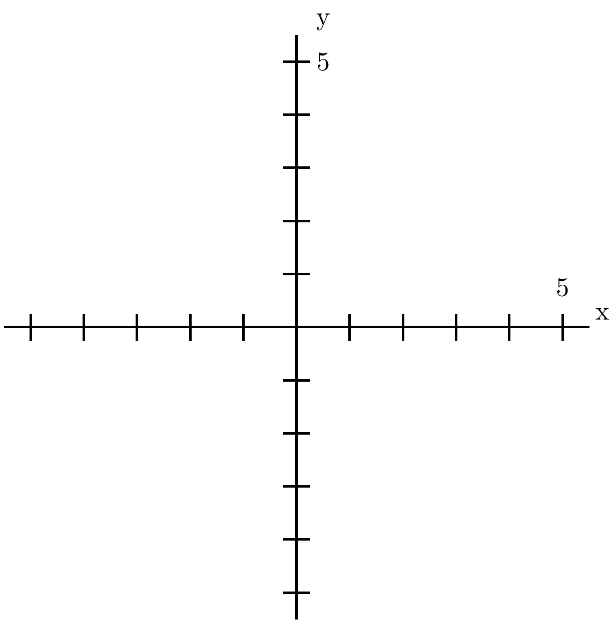
(a) $\lim_{x\to 3} \dfrac{x^2-4}{x-2} =$

(b) $\lim_{x\to 3} \dfrac{x^2-6x+9}{x^2-9} =$

3. (20 pts) Let $f(x) = x^3 - 3x^2 - 1$.

(a) Find and classify the extreme points (relative maximum and minimum points) of the graph of $y = f(x)$.

(b) Graph the given function $f(x)$, using your knowledge of calculus.



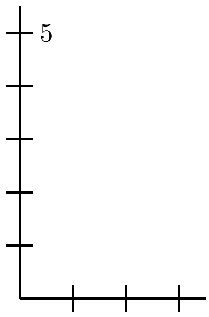
4. (30 pts) Find the following integrals.

(a) $\int_{-2}^{1} (x^2 + x + 1)\, dx =$

(b) $\int_{0}^{1} \frac{1}{x+3}\, dx =$

(c) $\int 3x^2 \sqrt{x^3 - 8}\, dx =$

5. (15 pts) Find the area bounded by the curves $y = x^2$ and $y = \frac{1}{x^2}$ and the vertical lines $x = 1$ and $x = 2$. (A sketch of the curves may help you.)



6. (15 pts) The demand equation for a certain product is $p = 256 - 50x$, where p is the price and x is the number of units produced. If the cost function is $C(x) = 182 + 56x$, determine the level of production that will maximize the profit.

7. (15 pts) A handmade box (with a top) is to have a square base and a volume of 32 cubic inches. The base costs $1.00 per square inch, the sides cost $4.00 per square inch, and the top costs $3.00 per square inch. What are the dimensions of the box that will minimize the cost?

8. (15 pts) The population of a colony of bacteria after t hours is given by $P(t) = 5000\, e^{.02t}$.

(a) Find the rate of growth of the population after 50 hours.

(b) In how many hours will the population double? (You may leave your answers in terms of e^x and $\ln x$.)

9. (15 pts) A rock is dropped from the top of a cliff 144 feet high. Its velocity at time t (in seconds) is $v(t) = -32t$ feet per second.

(a) Find $s(t)$, the height of the rock above ground.

(b) Find the time at which the rock hits the ground.

(c) Find the velocity with which the rock hits the ground.

10. (20 pts) Find the derivative of each of the following functions.

(a) $f(x) = (x^2 + 2)^3\sqrt{x^2 - 2x - 3}$

(b) $f(x) = \ln\left(\dfrac{xe^x}{\sqrt{1+x}}\right)$

11. (10 pts) Using the limit definition of the derivative (not the power formula), find $f'(x)$ if $f(x) = \dfrac{1}{x^2}$.