MATH 211 FINAL EXAM NAME ______________________

5/3/97 NO CALCULATORS! Show all necessary work. Be neat, clear, and brief.

1. (30 points) Find the derivative of each of these functions. You do not need to simplify your answers.

(a) $f(x) = x^2 + \dfrac{1}{x^2}$

(b) $f(x) = (x^5 + \ln x)^4$

(c) $f(x) = e^{x^2}\sqrt{x^3 + 5}$

(d) $f(x) = \dfrac{x^3 - 1}{x^2 + 1}$

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2. (15 pts) Find the following limits.

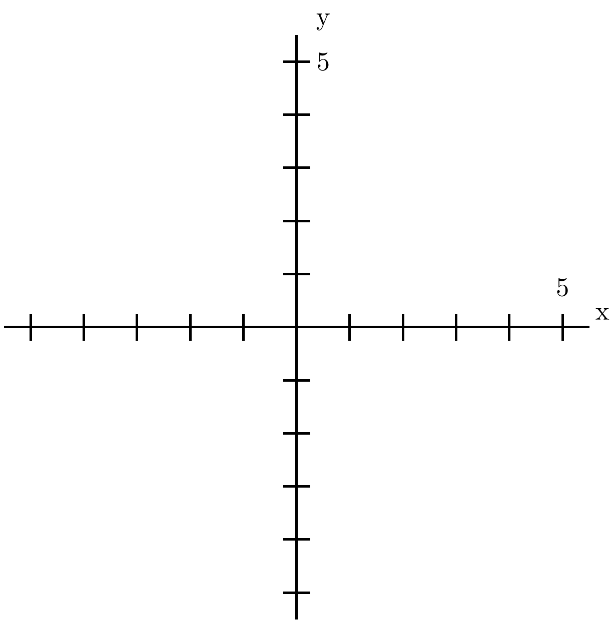
(a) $\lim_{x\to 6} \dfrac{x^2 - 4x - 12}{x - 6} =$

(b) $\lim_{x\to 3} \dfrac{x^2 - 4x - 12}{x^2 - 8x + 12} =$

3. (20 pts) Let $f(x) = \dfrac{1}{2}x + \dfrac{2}{x}$.

(a) Find and classify the extreme points (relative maximum and minimum points) of the graph of the given function $y = f(x)$.

(b) Graph the given function $f(x)$, using your knowledge of calculus. Be sure to find all asymptotes.



4. (7 pts) Let $P(t) = P_0 e^{kt}$. Find $P'(t)$ and check that $P'(t) = kP(t)$.

5. (8 pts) Find an equation for the line tangent to the curve $y = x \ln x$ at $x = 1$.

6. (15 pts) A store manager wants to build a 600-square-foot rectangular enclosure. Three sides of the enclosure will be built of redwood fencing, at a cost of \$14 per foot. The fourth side will be built of cement blocks, at a cost of \$28 per foot. Find the length and width of the enclosure that will minimize the total cost of the building materials.

7. (15 pts) Five grams of a certain radioactive material decays to 3 grams in 1 year. After how many years will just 1 gram remain?

8. (15 pts) The demand equation for a certain product is $p = 200 - 3x$, where p is the price and x is the number of units produced. The cost function is $C(x) = 75 + 80x - x^2$, where $0 \leq x \leq 40$.

(a) Determine the level of production that will maximize the profit, and determine the corresponding price.

(b) Suppose that the government imposes a tax of \$4 per unit produced. Find the new price that now maximizes the profit.

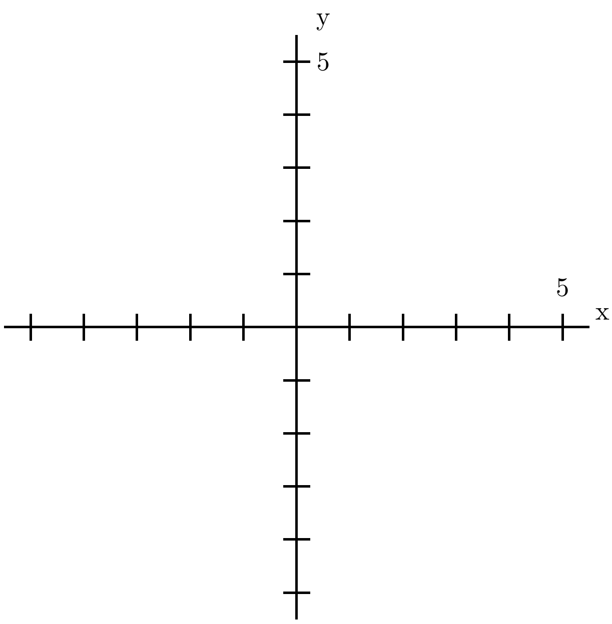
9. (25 pts) Find the following integrals.

(a) $\int \left(4x^3 - 2 + \frac{1}{x}\right)\, dx =$

(b) $\int_1^4 \frac{1}{x^2}\, dx =$

(c) $\int t^2 e^{t^3}\, dt =$

10. (15 pts) Find the area bounded by the curves $y = 2x^2 + x - 5$ and $y = x + 3$ (from $x = -2$ to $x = 2$). First graph the two functions.



11. (25 pts)

(a) $f(x) = \sqrt{x + \sqrt{x}} =$

$f'(x) =$

(b) $f(x) = \ln\left((x^2+3)^5(x^3+1)^{-4}\right) =$

$f'(x) =$

(c) $\displaystyle\int \frac{x^2 - 2x}{x^3 - 3x^2 + 1}\, dx =$

12. (10 pts) Using the limit definition of the derivative (not the power formula), find $f'(x)$ if $f(x) = x^2 + 5x$.