MATH 211 TEST III NAME ____________________

Prof. J. Beachy 4/11/97 Circle recitation time: T 11 Th 10 Th 11

NO CALCULATORS! Be sure to show all necessary work.

1. (5 pts) Solve this equation for x: $4 - 3e^{x+5} = 0$

2. (20 pts) Find the derivative of each of these functions.

(a) $f(x) = e^{-x} \ln x$

$f'(x) =$

(b) $f(x) = \left(e^{x^4} + x^3\right)^5$

$f'(x) =$

(c) $f(x) = \dfrac{e^{3x}}{x^2 + 1}$

$f'(x) =$

(d) $f(x) = \ln\left(\dfrac{e^x \sqrt{x^4 + 1}}{(x^2 - 2x + 3)^5}\right) =$

$f'(x) =$

3. (10 pts) For a certain radioactive substance, the weight in grams after t years is given by $P(t)$. The function $P(t)$ has these properties: $P'(t) = -.08P(t)$ and $P(0) = 30$.

Find the formula for $P(t)$, and then find $P(10)$.

4. (15 pts) The growth rate of a certain bacteria culture is proportional to its size. If the bacteria culture doubles in size every 20 minutes, how long will it take for the culture to increase to 10 times its initial size?

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TOTAL	/ 100
GRADE	
Quizes	
Avg	

5. (7 pts) Check this answer by differentiating: $\int 6x(x^2+1)^2\,dx = (x^2+1)^3 + C$

6. (18 pts) Find these antiderivatives:

(a) $\int (4x^3 + 8x^2 + 5)\,dx =$

(b) $\int e^{2x}\,dx =$

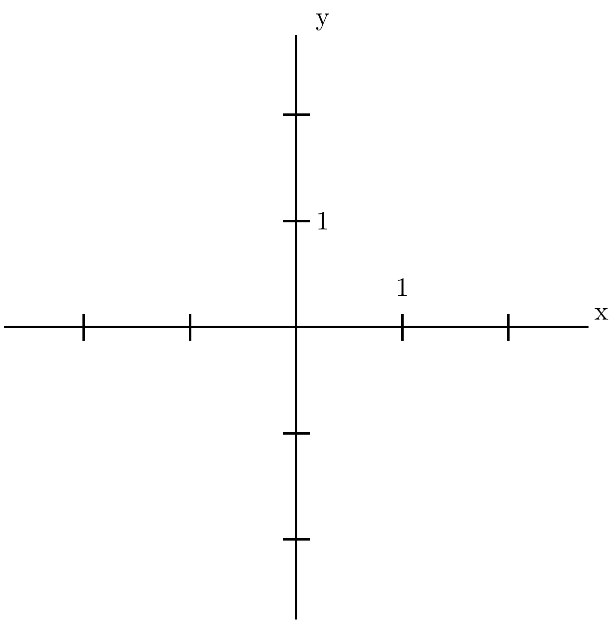
(c) $\int \left(\frac{1}{\sqrt{x}} - 2\sqrt{x}\right)\,dx =$

7. (7 points) Find the equation of the tangent line to the curve $y = \ln x + \frac{2}{x}$ at $x = 1$.

8. Use calculus techniques to sketch the graph of $y = xe^x$.

(a) (12 pts) Find the coordinates of the extreme points of the curve. Take the second derivative and determine where the curve is concave up and where it is concave down.

(b) (6 pts) Graph the function. Hint: there is a horizontal asymptote.
You can use the approximate values $e \sim 2.7$, $e^2 \sim 7.2$, $e^{-1} \sim .37$, $e^{-2} \sim .14$, $e^{-3} \sim .05$.



MATH 211, 4/21/97 TEST III SOLUTIONS Prof. J. Beachy

1. (5 pts) Solve this equation for x: $4 - 3e^{x+5} = 0$

$4 = 3e^{x+5}$ $4/3 = e^{x+5}$ $\ln(4/3) = x + 5$ $x = -5 + \ln(4/3)$

2. (20 pts) Find the derivative of each of these functions.

(a) $f(x) = e^{-x} \ln x$ Use the product rule: $f'(x) = e^{-x}(-1) \cdot \ln x + e^{-x} \cdot \frac{1}{x}$

(b) $f(x) = \left(e^{x^4} + x^3\right)^5$ Use the general power formula: $f'(x) = 5(e^{x^4} + x^3)^4(e^{x^4}(4x^3) + 3x^2)$

(c) $f(x) = \dfrac{e^{3x}}{x^2+1}$ Use the quotient rule: $f'(x) = \dfrac{3e^{3x}(x^2+1) - e^{3x}(2x)}{(x^2+1)^2}$

(d) $f(x) = \ln\left(\dfrac{e^x\sqrt{x^4+1}}{(x^2-2x+3)^5}\right) = \ln(e^x) + \ln(\sqrt{x^4+1}) - \ln(x^2-2x+3)^5 = x + \frac{1}{2}\ln(x^4+1) - 5\ln(x^2-2x+3)$

$f'(x) = 1 + \frac{4x^3}{2(x^4+1)} - \frac{5(2x-2)}{x^2-2x+3}$ You can see it pays to simplify $f(x)$ first.

3. (10 pts) For a certain radioactive substance, the weight in grams after t years is given by $P(t)$. The function $P(t)$ has these properties: $P'(t) = -.08P(t)$ and $P(0) = 30$.

The formula for $P(t)$ is $P(t) = 30e^{-.08t}$, and $P(10) = 30e^{-.8}$.

4. (15 pts) The growth rate of a certain bacteria culture is proportional to its size. If the bacteria culture doubles in size every 20 minutes, how long will it take for the culture to increase to 10 times its initial size?

The solution is $P(t) = P_0e^{kt}$, and when $t = 20$ we have $2P_0 = P(20) = P_0e^{k(20)}$, so $2 = e^{20k}$, and therefore $k = (\ln 2)/20$.

We then need to solve the equation $P(t) = 10P_0$, or $P_0e^{kt} = 10P_0$. This gives the equation $e^{kt} = 10$, so $t = (\ln 10)/k$, or $t = (20\ln 10)/\ln 2$.

5. (7 pts) Check this answer by differentiating: $\int 6x(x^2+1)^2\,dx = (x^2+1)^3 + C$

Solution: $\frac{d}{dx}((x^2+1)^3 + C) = 3(x^2+1)^2(2x) = 6x(x^2+1)^2$. To get full credit, you *must* show that you differentiated the right hand side of the equation.

6. (18 pts) Find these antiderivatives:

(a) $\int (4x^3 + 8x^2 + 5)\,dx = x^4 + \dfrac{8}{3}x^3 + 5x + C.$ (b) $\int e^{2x}\,dx = \dfrac{1}{2}e^{2x} + C.$

(c) $\int \left(\dfrac{1}{\sqrt{x}} - 2\sqrt{x}\right)dx = \int (x^{-1/2} - 2x^{1/2})\,dx = 2x^{1/2} - \dfrac{4}{3}x^{3/2} + C$

7. (7 points) Find the equation of the tangent line to the curve $y = \ln x + \dfrac{2}{x}$ at $x = 1$.

We have $y' = \frac{1}{x} - \frac{2}{x^2}$, so when $x = 1$, $y' = 1 - 2 = -1$ and $y = \ln 1 + 2 = 2$. The equation of the tangent line is $y = (-1)(x-1) + 2$.

8. Use calculus techniques to sketch the graph of $y = xe^x$.

(a) (12 pts) Find the coordinates of the extreme points of the curve. Take the second derivative and determine where the curve is concave up and where it is concave down.

We have $y' = e^x + xe^x = (1+x)e^x$ and $y'' = e^x + e^x + xe^x = (2+x)e^x$. Thus $y' = 0$ for $x = -1$, and y'' is positive at $x = -1$, so this is a relative minimum. Since y'' is negative for $x < -2$ and positive for $x > -2$, the curve is concave down for $x < -2$ and concave up for $x > -2$.

(b) (6 pts) Graph the function. *Solution:* Using the approximate values that are given, these points are on the curve: $(2, 14.4)$, $(1, 2.7)$, $(0, 0)$, $(-1, -.37)$, $(-2, -.28)$, $(-3, -.15)$. The x-axis is a horizontal asymptote. The point $(-1, -.37)$ is a relative minimum point, and $(-2, -.28)$ is a point of inflection.