MATH 211, Section C TEST I NAME SOLUTIONS
Prof. J. Beachy 9/20/02 Circle recitation time: T 10:00 T 11:00 Th 11:00
NO CALCULATORS! Be sure to show all necessary work.

1. (7 pts; p 38 #37) Find the domain of $f(x) = \dfrac{x^2 - 3x}{|4x - 7|}$.

The only problem with the formula is division by zero when $|4x - 7| = 0$. That means that the domain is all real numbers except $x = \dfrac{7}{4}$. The answer can also be given as $\{x \in \mathbf{R} \mid x \neq \dfrac{7}{4}\}$ or $(-\infty, \frac{7}{4}) \bigcup (\frac{7}{4}, +\infty)$.

2. (8 pts; p 51 #27) Find the slope of the line containing the points $(\frac{2}{5}, \frac{1}{2})$ and $(-3, \frac{4}{5})$.

$$m = \frac{\frac{4}{5} - \frac{1}{2}}{-3 - \frac{2}{5}} = \frac{(\frac{4}{5} - \frac{1}{2})(10)}{(-3 - \frac{2}{5})(10)} = \frac{8 - 5}{-30 - 4} = -\frac{3}{34}$$

3. (10 pts; p 63 Ex 6) On the axes below, graph $3x + 3y = -6$ and $y = \dfrac{1}{x}$.

See Example 6 on page 63 for the graph of $y = \frac{1}{x}$. The equation $3x + 3y = -6$ simplifies to $y = -x - 2$, in slope-intercept form. (This happens to be the line tangent to the graph at $(-1, -1)$.)

4. (15 pts; p 148 #2,13,20) Find the derivative of each of the following functions. Use the power rule–do *not* use the limit definition of the derivative.

(a) If $f(x) = x^8$, then $f'(x) = 8x^7$

(b) $\dfrac{d}{dx}\left(\dfrac{1}{2}x^{4/5}\right) = \dfrac{1}{2} \cdot \dfrac{4}{5}x^{-1/5} = \dfrac{2}{5}x^{-1/5}$

(c) If $f(x) = \sqrt[5]{x} - \dfrac{2}{x} = \quad x^{1/5} - 2x^{-1}$ then $f'(x) = \dfrac{1}{5}x^{-4/5} + 2x^{-2}$

5. (10 pts) (a) Find an equation of the tangent line to the graph of $y = \dfrac{1}{x^2}$ at $(2, \dfrac{1}{4})$.

Using the power rule for $y = x^{-2}$, we get $\dfrac{dy}{dx} = -2x^{-3}$.
When $x = 2$, the derivative is $\dfrac{-2}{8} = -\dfrac{1}{4}$, so the tangent line at $(2, \frac{1}{4})$ is $y = -\frac{1}{4}(x - 2) + \frac{1}{4}$.

(b) Find the point on the graph of $y = x^2$ where the tangent line has the same slope as the line $2x - y = 4$.

The equation $2x - y = 4$ can be written as $y = 2x - 4$ (in slope-intercept form), so the line has slope 2. This means that we need to solve the equation $f'(x) = 2$, where $f(x) = x^2$.
We get the equation $2x = 2$, so $x = 1$, and the corresponding point on the graph of $y = x^2$ is $(1, 1)$.

6. (10 pts) Graph the function $f(x) = \begin{cases} x^2 - 1 & \text{for } x < 1 \\ 2x - 2 & \text{for } x \geq 1 \end{cases}$ Is $f(x)$ continuous at $x = 1$? Explain.

The first part of the graph (when $x < 1$) is a parabola, and the second part (when $1 \leq x$) is a line.
The function is continuous at $x = 1$ because the two parts of the graph meet at $(1, 0)$.

7. (15 pts) Find these limits (using the algebraic method).

$$\lim_{x\to 4}\frac{x^2-5x+4}{x^2-3x-4}=\lim_{x\to 4}\frac{(x-4)(x-1)}{(x-4)(x+1)}=\lim_{x\to 4}\frac{x-1}{x+1}=\frac{4-1}{4+1}=\frac{3}{5}$$

Note: You should first substitute $x=4$. When you get $\frac{0}{0}$ you know (from high school algebra) that $x-4$ is a factor of both the numerator and denominator. That makes it easy to find the other factors.

$$\lim_{x\to -1}\frac{4x^2+5x-7}{3x^2-2x+1}=\lim_{x\to -1}\frac{4(-1)^2+5(-1)-7}{3(-1)^2-2(-1)+1}=\frac{4-5-7}{3+2+1}=\frac{-8}{6}=\frac{-4}{3}$$

$\lim_{x\to 0^-}\frac{|x|}{x}=\lim_{x\to 0^-}\frac{-x}{x}=\lim_{x\to 0^-}-1=-1$ because $x\to 0^-$ means that $x<0$, and in this case $|x|=-x$.

8. (25 pts; p 137 #11, 29)

(a) Complete the limit definition of the derivative of a function $f(x)$: $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

(b) **Use the limit definition** of the derivative of a function to find $f'(x)$, for the function $f(x)=x^2+x$.

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{((x+h)^2+(x+h))-(x^2+x)}{h}=\lim_{h\to 0}\frac{x^2+2xh+h^2+x+h-x^2-x}{h}$$

$$=\lim_{h\to 0}\frac{2xh+h^2+h}{h}=\lim_{h\to 0}\frac{(h)(2x+h+1)}{h}=\lim_{h\to 0}(2x+h+1)=2x+(0)+1=2x+1$$

(c) **Use the limit definition** of the derivative of a function to find $f'(x)$, for the function $f(x)=\frac{1}{x^2}$.

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{1}{h}\left(\frac{1}{(x+h)^2}-\frac{1}{x^2}\right)=\lim_{h\to 0}\frac{1}{h}\left(\frac{(1)(x^2)}{(x+h)^2(x^2)}-\frac{(x+h)^2(1)}{(x+h)^2(x^2)}\right)$$

$$=\lim_{h\to 0}\frac{1}{h}\left(\frac{x^2-(x^2+2xh+h^2)}{(x+h)^2(x^2)}\right)=\lim_{h\to 0}\frac{1}{h}\left(\frac{-2xh-h^2}{(x+h)^2(x^2)}\right)=\lim_{h\to 0}\frac{(h)(-2x-h)}{(h)(x+h)^2(x^2)}$$

$$=\lim_{h\to 0}\frac{-2x-h}{(x+h)^2(x^2)}=\frac{-2x-0}{(x+0)^2(x^2)}=\frac{-2x}{(x^2)(x^2)}=\frac{-2}{x^3}$$

Note: When you use the limit definition, keep the limit notation until you make the necessary substitution. Also, since you know the power formula, you can use it to check your answers to (b) and (c).

The class average on the test was 72.9.

Grading scale: 90–95 A (11); 80–89 B (26); 65–79 C (30); 55–64 D (8); 31–54 F (14)