

1. (10 pts; p262 #27) Raggs, Ltd., a clothing firm, determines that in order to sell x suits, the price per suit should be $p = 150 - 0.5x$. It also determines that the total cost of producing x suits is given by $C(x) = 4000 + 0.25x^2$.

Find the function $P(x)$ that gives the profit as a function of x .

The profit made is just the amount of money taken in, minus the costs. That is, the profit is equal to revenue minus cost. The total amount of money that the clothing firm takes in is equal to the price per suit multiplied by the number of suits that are sold.

We need to maximize the profit.

Profit = price times #suits minus cost or $P(x) = p(x) \cdot x - C(x)$

$$P(x) = (150 - 0.5x)x - (4000 + 0.25x^2) = 150x - 0.5x^2 - 4000 - 0.25x^2 = 150x - 4000 - 0.75x^2$$

Answer: $P(x) = 150x - 4000 - 0.75x^2$

2. (10 pts; p 263 #33) A rectangular box with a volume of 350 ft^3 is to be constructed with a square base and top. The cost per square foot for the bottom is 15 cents, for the top is 10 cents, and for the sides is 2.5 cents. What dimensions will minimize the cost?

Find the function to be minimized (but do not solve further).

Don't be intimidated by a problem like this. It just takes some common sense. The costs are different for the different parts of the box, so we need to look at these separately: the bottom, the top, each of the four sides. Let's find the cost in dollars instead of cents. The total cost for the bottom is 0.15 times the number of square feet in the bottom; the total cost for the top is 0.1 times the number of square feet in the top (which is the same as the bottom); the total cost for the sides is 0.025 times the area of each side, multiplied by four. Finally, we need to add up these costs.

Refer to the diagram on page 263 of the text. I thought it was fair to expect you to draw your own diagram since I had done the problem in class. Note: in writing out the problem, by mistake I copied the volume as 350 ft^3 instead of 320 ft^3 . Luckily, I didn't ask you to solve the problem completely.

We need to minimize the total cost.

Solution: Let x be the length of one side of the base, and let y be the height of the box.

The cost of the base is $0.15x^2$.

The cost of the top is $0.1x^2$.

The cost of the sides is $0.025xy$, multiplied by 4, giving $0.1xy$.

Total cost: $C(x) = 0.15x^2 + 0.1x^2 + 0.1xy = 0.25x^2 + 0.1xy$.

Now there is a problem: $C(x)$ isn't written as a function of x , since it has a y in it. Before we can use calculus, we need to substitute for y , so that everything is expressed in terms of x .

The problem states that the volume of the box is 350 ft^3 , so with a length of x , width of x , and height of y we get the equation $x^2y = 350$. Solving for y we get $y = \frac{350}{x^2}$. Finally, we need to substitute and simplify.

$$\text{Total cost: } C(x) = 0.25x^2 + 0.1xy = 0.25x^2 + 0.1x \left(\frac{350}{x^2} \right) = 0.25x^2 + \frac{35}{x}.$$

Answer: $C(x) = 0.25x^2 + \frac{35}{x}$ Alternate solution: the cost in cents is $C(x) = 25x^2 + \frac{3500}{x}$.