MATH 211 QUIZ 9 NAME ________________________________

Prof. J. Beachy Friday, 11/8/02 Circle C1(T10) C2(T11) C3(Th11)

These are the new formulas you need to know, in addition to the product rule, quotient rule, and extended power rule. Also remember the chain rule: $\frac{d}{dx} f(u(x)) = f'(u(x))\, u'(x)$

Exponentials: $\frac{d}{dx} e^x = e^x$ $\quad \frac{d}{dx} e^{u(x)} = e^{u(x)}\, u'(x)$ or $\frac{d}{dx} e^{u(x)} = u'(x) e^{u(x)}$

Natural logs: $\frac{d}{dx} \ln x = \frac{1}{x}$ $\quad \frac{d}{dx} \ln u(x) = \frac{1}{u(x)}\, u'(x)$ or $\frac{d}{dx} \ln u(x) = \frac{u'(x)}{u(x)}$

You should also remember these useful rules for exponents and natural logs:

$e^{u(x)} e^{v(x)} = e^{u(x)+v(x)} \qquad \left(e^{u(x)}\right)^k = e^{ku(x)}$

$\ln\left(u(x)v(x)\right) = \ln u(x) + \ln v(x) \qquad \ln\left(\frac{u(x)}{v(x)}\right) = \ln u(x) - \ln v(x) \qquad \ln u(x)^k = k \ln u(x)$

1. (14 pts) Find these derivatives:

(a) (4.1 #57) $\frac{d}{dx}\left(e^{3x}+1\right)^5 = 5(e^{3x}+1)^4 \cdot \frac{d}{dx} e^{3x} = 5(e^{3x}+1)^4 \cdot e^{3x} \cdot 3 = 15 e^{3x}\left(e^{3x}+1\right)^4$

(b) (4.1 #63) $\frac{d}{dx}\left(e^{\sqrt{x}} + \sqrt{e^x}\right) = \frac{d}{dx}\left(e^{x^{1/2}} + (e^x)^{1/2}\right) = \frac{d}{dx} e^{x^{1/2}} + \frac{d}{dx} e^{\frac{1}{2}x}$

$= e^{x^{1/2}} \cdot \frac{1}{2} x^{-1/2} + e^{\frac{1}{2}x} \cdot \frac{1}{2} = \frac{e^{\sqrt{x}}}{2\sqrt{x}} + \frac{1}{2} e^{\frac{1}{2}x}$

(c) (4.2 #53) $\frac{d}{dx}\left(\ln(\ln 4x)\right) = \frac{1}{\ln 4x} \cdot \frac{d}{dx} \ln 4x = \frac{1}{\ln 4x} \cdot \frac{1}{4x} \cdot 4 = \frac{1}{x \ln 4x}$

(d) (4.2 #84) $\frac{d}{dx}\left(\ln\sqrt{5+x^2}\right) = \frac{d}{dx}\left(\ln(5+x^2)^{1/2}\right) = \frac{d}{dx}\left(\frac{1}{2}\ln(5+x^2)\right) = \frac{1}{2} \cdot \frac{1}{5+x^2} \cdot 2x = \frac{x}{5+x^2}$

2. (6 pts; 4.3 #24) The cost of a first-class stamp in 1962 was 4 cents. In 1999, the cost was 33 cents. Assume that this increase can be described as exponential growth.

(a) Write down the equation for exponential growth.

$P(t) = P_0 e^{kt}$

(b) Find the constants in your equation for exponential growth. (Write an equation for the growth rate k in terms of the given information, and solve for k, but do not try to get a numerical answer, which would require a calculator.)

Let $t = 0$ in 1962. Then $P_0 = 4$, and we have the equation $P(t) = 4e^{kt}$ as the first step.

In 1999 the cost was 33 cents, and $t = 1999 - 1962 = 37$. Substitute these values into the equation:

$33 = 4e^{k \cdot 37} \qquad \frac{33}{4} = e^{k \cdot 37} \qquad \ln\left(\frac{33}{4}\right) = \ln\left(e^{k \cdot 37}\right) = k \cdot 37 \qquad k = \frac{1}{37} \ln\left(\frac{33}{4}\right)$