MATH 211 C (J. Beachy) QUIZ 7 SOLUTIONS 3/28/03

1. (6 pts) Find $f'(x)$ for $f(x) = x^4 e^{x^2+1}$.

$f'(x) = 4x^3 \cdot e^{x^2+1} + x^4 \cdot e^{x^2+1}(2x)$ *Optional simplification:* $f'(x) = (4x^3 + 2x^5)e^{x^2+1}$

2. (6 pts; p299 #63) Find $f'(x)$ for $f(x) = e^{\sqrt{x}} + \sqrt{e^x}$.

Simplify first: $f(x) = e^{\sqrt{x}} + \sqrt{e^x} = e^{x^{1/2}} + (e^x)^{1/2} = e^{x^{1/2}} + e^{(1/2)x}$

$f'(x) = e^{x^{1/2}}(1/2)x^{-1/2} + e^{(1/2)x}(1/2)$ *Optional simplification:* $f'(x) = \dfrac{e^{\sqrt{x}}}{2\sqrt{x}} + \dfrac{1}{2}\sqrt{e^x}$

3. (8 pts) For the function $f(x) = xe^{2x}$, find the absolute maximum and absolute minimum values on the interval $[-1, 0]$.

We need to test the values of $f(x)$ at the critical points and end points. To find $f'(x)$, we need to use the product rule.

$f'(x) = 1 \cdot e^{2x} + x \cdot e^{2x}(2) = e^{2x} + 2xe^{2x} = (1 + 2x)e^{2x}$ Now set $f'(x) = 0$.

$(1 + 2x)e^{2x} = 0$ $1 + 2x = 0$ $x = -1/2$ *We can cancel the term e^{2x}, since it is never zero.*

$f(-1) = (-1)e^{2(-1)} = -\dfrac{1}{e^2}$ $f(-1/2) = (-1/2)e^{2(-1/2)} = (-1/2)e^{-1} = -\dfrac{1}{2e}$ $f(0) = (0)e^{2(0)} = 0$

We have $2 < e$ $2e < e^2$ $\dfrac{1}{2e} > \dfrac{1}{e^2}$ $-\dfrac{1}{2e} < -\dfrac{1}{e^2}$ Therefore $f(-1/2) < f(-1) < f(0)$.

Answer: $f(-1/2) = -\dfrac{1}{2e}$ is the minimum value of $f(x)$ on $[-1, 0]$ and $f(0) = 0$ is the maximum.

MATH 211 C (J. Beachy) QUIZ 8 SOLUTIONS 4/4/03

1. (6 pts; p316 #83) Find $f'(x)$ for $f(x) = \ln\left[\dfrac{x^5}{(8x+5)^2}\right]$. *Use the identities on the top of page 305.*

$f(x) = \ln\left[\dfrac{x^5}{(8x+5)^2}\right] = \ln[x^5] - \ln[(8x+5)^2] = 5\ln x - 2\ln(8x+5)$

$f'(x) = 5 \cdot \dfrac{1}{x} - 2 \cdot \dfrac{8}{8x+5} = \dfrac{5}{x} - \dfrac{16}{8x+5}$

2. (6 pts; p316 #68) The demand function for a certain product is $D(p) = 800e^{-0.125p}$, where p is the price per unit. Find the value of p for which the revenue function $R(p) = pD(p)$ will be a maximum.

This problem was not on the list of assigned problems, although the adjacent problems #67 and #69 were on the list. Demand functions were introduced on page 69; you don't really need to know anything about them because the formula for the revenue function is given in the problem.

$R(p) = p \cdot D(p) = p \cdot 800e^{-0.125p} = 800pe^{-0.125p}$

To find the maximum value for $R(p)$, we need to set $R'(p) = 0$. Finding $R'(p)$ requires the product rule, since $R(p)$ is the product of $800p$ and $e^{-0.125p}$.

$R'(p) = 800 \cdot e^{-0.125p} + 800p \cdot e^{-0.125p}(-0.125) = 800e^{-0.125p} - 100pe^{-0.125p}$ Now solve $R'(p) = 0$.

$800e^{-0.125p} - 100pe^{-0.125p} = 0$ $(800 - 100p)e^{-0.125p} = 0$ $800 - 100p = 0$ $800 = 100p$ $p = 8$

The answer $p = 8$ does give a maximum value, since $R'(7) > 0$ and $R'(9) < 0$.

3. (8 pts; p327 #3) The marketing manager for a national hamburger firm estimates that the number of their franchises will grow exponentially at the rate of 10% per year, following the model $P(t) = 50e^{0.10t}$. Here $P(t)$ is the number of franchises at time t, and t is measured in years.

(a) How many franchises will there be in 20 years? $P(20) = 50e^{0.10(20)} = 50e^2$

(b) How many years will it take for the initial number of 50 franchises to double?

Solve the equation $P(t) = 100$ for t. $50e^{0.10t} = 100$ $e^{0.10t} = 2$ $.10t = \ln 2$ $t = 10\ln 2$