MATH 211 QUIZ 9 Solutions 4/11/03

1. (5 pts; p331 #39) The population of fish in a lake is given by the following equation (the time t is given in months).

$$P(t) = \frac{2500}{1 + 5.25e^{-0.32t}}$$

(a) Find the population after 0 months.

$$P(0) = \frac{2500}{1 + 5.25e^{-0.320}} = \frac{2500}{6.25} = 400$$

(b) Find the rate of change $P'(t)$.

Using the quotient rule, we get the following derivative. (Remember that $\frac{d}{dt}e^{u(t)} = e^{u(t)}u'(t)$.)

$$P'(t) = \frac{-2500(5.25e^{-0.32t})(-.32)}{(1 + 5.25e^{-0.32t})^2} = \frac{4200e^{-0.32t}}{(1 + 5.25e^{-0.32t})^2}$$

2. (5 pts; p371 #19) $\int \sqrt{x}\, dx = \int x^{1/2}\, dx = \frac{x^{3/2}}{3/2} + C = \frac{2}{3}x^{3/2} + C$

3. (5 pts; p371 #31) A company determins that the marginal cost C' of producing the xth unit of a certain product is given by

$$C'(x) = x^3 - 2x$$

Find the total-cost function $C(x)$, assuming fixed costs to be \$100.

Since we are given the derivative $C'(x)$, we need to find the general antiderivative. Because the cost function already uses C, we should use another letter, say K, to denote the constant. Then the fixed costs give the cost of producing 0 units, so we can set $C(0) = 100$ to find the constant K.

$$C(x) = \frac{x^4}{4} - x^2 + K$$

When $x = 0$, $100 = C(0) = \frac{0^4}{4} - 0^2 + K$ $K = 100$

$$C(x) = \frac{1}{4}x^4 - x^2 + 100$$

4. (5 pts) Newton's law of cooling

(a) Find $f'(x)$ for $f(x) = C + ae^{-kx}$, where C, a, and k are constants.

$$f'(x) = 0 + ae^{-kx}(-k) = -kae^{-kx}$$

(b) For $f(x) = C + ae^{-kx}$, find $-k(f(x) - C)$ and show that this is the same as your answer in part (a).

$$-k(f(x) - C) = -k(ae^{-kx}) = -kae^{-kx} = f'(x)$$