MATH 211, Section C — Review for EXAM 3 — 4/23/2003

Professor John Beachy — Bring a valid picture I.D.

Exam 3, covering Sections 4.1–4.4 and 5.1–5.3, has been postponed to **Wednesday, April 23**.

Sections 4.1–4.4

The derivative of $f(x) = a^x$ is $f'(x) = ka^x$, where $k = f'(0)$. If we choose the base e, then $f(x) = e^x$ has derivative $f'(x) = e^x$, because $f'(0) = 1$. That is the reason for using $e = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = 2.7182818.....$ The connection between the exponential function e^x and the natural logarithm function $\ln x$ is given by the two equations $e^{\ln x} = x$ and $\ln e^x = x$, which express the fact that the functions are inverses of each other. This means that their graphs are symmetric about the line $y = x$ (see the important graph at the bottom of page 307). Facts to know:

$$\ln 1 = 0 \qquad \ln e = 1 \qquad e^{\ln x} = x \qquad \ln\left(e^{u(x)}\right) = u(x) \qquad e^{u(x)}e^{v(x)} = e^{u(x)+v(x)} \qquad \left(e^{u(x)}\right)^k = e^{ku(x)}$$

$$\ln u(x)^k = k\ln u(x) \qquad \ln\left(u(x)v(x)\right) = \ln u(x) + \ln v(x) \qquad \ln\left(\frac{u(x)}{v(x)}\right) = \ln u(x) - \ln v(x) \qquad \text{(see p 305)}$$

Essential differentiation formulas: (also review the product rule and quotient rule)

Chain rule: $\frac{d}{dx} f(u(x)) = f'(u(x))\,u'(x)$ Extended power rule: $\frac{d}{dx}(u(x))^k = k(u(x))^{k-1}u'(x)$

Exponentials: $\frac{d}{dx} e^x = e^x$ $\frac{d}{dx} e^{u(x)} = e^{u(x)}\,u'(x)$ Natural logs: $\frac{d}{dx}\ln x = \frac{1}{x}$ $\frac{d}{dx}\ln u(x) = \frac{1}{u(x)}\,u'(x)$

Applications: The model for "uninhibited growth" (see p 318) is based on the assumption that the rate of growth is proportional to the amount, at any time. That is, $\frac{dP}{dt} = kP(t)$, at time t. The solution is $P(t) = P_0e^{kt}$, where P_0 is the initial amount (when $t = 0$). The problems usually have enough information to find P_0 and k, and then you can answer additional questions about the situation. The same equation is used for radioactive decay (in this case k is negative). Sometimes the half-life is given, and this can be used to find the decay constant k.

Sections 5.1–5.3

In the earlier chapters we learned how to find the rate of growth of a function (its derivative) when we were given a formula for the function. In this chapter, the situation is reversed: Suppose that we know the formula for the rate of growth of a function. Can we find a formula for the function?

We use the notation $\int f(x)\,dx$ for the general *antiderivative* of $f(x)$, and so

$$\int f(x)\,dx = F(x) + C \qquad \text{means} \qquad F'(x) = f(x)\,.$$

With this notation (also called an *indefinite integral*), each of the differentiation formulas has a corresponding integration formula. The rules "the derivative of a sum is the sum of the derivatives" and "the derivative of a constant times a function is the constant times the derivative" give us these helpful rules.

$$\int [f(x)+g(x)]\,dx = \int f(x)\,dx + \int g(x)\,dx \qquad \int kf(x)\,dx = k\int f(x)\,dx \qquad (k \text{ any constant})$$

The basic differentiation formulas $\frac{d}{dx}x^n = nx^{n-1}$ $\frac{d}{dx}e^x = e^x$ $\frac{d}{dx}\ln x = \frac{1}{x}$ give us these formulas.

$$\int u^n\,du = \frac{u^{n+1}}{n+1} + C \quad (n \neq -1) \qquad \int e^u\,du = e^u + C \qquad \int \frac{1}{u}\,du = \ln|u| + C$$

The author makes this definition: $\int_a^b f(x)\,dx = F(b) - F(a)$, where $F(x)$ is any antiderivative of $f(x)$. You should think of this *definite integral* as an averaging process:

$\int_a^b f(x)\,dx$ equals the width $(b-a)$ of the interval multiplied by the average height of the function $f(x)$ on $[a, b]$.

This is useful in finding the area below a curve, or in finding a total output when you know the rate of output. (See the author's definition of the average value of a function, on page 394.) You need to know how to approximate a definite integral, as in Exercises 1 and 2 on page 396 (a problem on the exam would have fewer subdivisions since you can't use a calculator).

Review problems

4.1 #21,29,59,61; 4.2 #23–28,51,55,57,79,89,91; 4.3 #17,33; 4.4 #7,25,31,35;

5.1 #21,23,25,35,45,46; 5.2 #5,17,63,69; 5.3 #1,13,17