MATH 211, Section C — Review for EXAM 2 — 10/14/2002

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Exam 2 is scheduled for **Friday, October 18**. It will cover Sections 2.5–2.9 and 3.1–3.3. Bring a valid picture I.D.

Sections 2.5–2.9

Basic differentiation formulas: $\frac{d}{dx}(f(x)+g(x)) = f'(x)+g'(x)$ $\quad \frac{d}{dx}\, cf(x) = cf'(x)$ $\quad \frac{d}{dx}\, x^k = kx^{k-1}$

Product rule: $\frac{d}{dx}\, f(x)g(x) = f'(x)g(x) + f(x)g'(x)$ Quotient rule: $\frac{d}{dx}\,\frac{N(x)}{D(x)} = \frac{N'(x)D(x)-N(x)D'(x)}{D(x)^2}$

Extended power rule: $\frac{d}{dx}(u(x))^k = k(u(x))^{k-1}u'(x)$ Chain rule: $\frac{d}{dx}\, f(u(x)) = f'(u(x))\,u'(x)$

Higher order derivatives use the notation $f''(x)$, $f'''(x)$, etc., or $\frac{d^2y}{dx^2}$, $\frac{d^3y}{dx^3}$, etc.

The tangent line to the curve $y = f(x)$ at the point (a, b) is $y = m(x-a)+b$, where $m = f'(a)$ and $b = f(a)$.

Interpreting the first derivative: the derivative of a function measures its rate of growth (at a point), which is equal to the slope of its graph at that point, and also equal to the slope of the tangent line at that point.

If the derivative of $y = f(x)$ is positive, then the tangent line has positive slope, and y is increasing.

If the derivative of $y = f(x)$ is negative, then the tangent line has negative slope, and y is decreasing.

In physics, if $y = s(t)$ gives the distance of an object from its starting point at time t, then $s'(t)$ is the velocity $v(t)$ of the object, and $s''(t) = v'(t)$ is its acceleration.

In economics, marginal cost, marginal revenue, and marginal profit are simply the derivatives of the cost function, the revenue function, and the profit function (respectively).

Sections 3.1–3.3

The main idea is to use the sign of the first derivative and the sign of the second derivative of a function to provide useful information about the graph of the function.

$f'(x) > 0 \quad \longleftrightarrow \quad$ positive slope $\quad \longleftrightarrow \quad f(x)$ is increasing

$f'(x) < 0 \quad \longleftrightarrow \quad$ negative slope $\quad \longleftrightarrow \quad f(x)$ is decreasing

$f''(x) > 0 \quad \longleftrightarrow \quad f'(x)$ is increasing $\quad \longleftrightarrow \quad f(x)$ is concave up

$f''(x) < 0 \quad \longleftrightarrow \quad f'(x)$ is decreasing $\quad \longleftrightarrow \quad f(x)$ is concave down

These correspondences allow us to find relative maximum and relative minimum values of a function, and provide help in graphing because they say something about the shape of the graph of a function.

In Sections 3.1–3.3 you should review the definitions and theorems (look for the boxes of text that are shaded in grey). These include the definitions of *increasing* and *decreasing* (p 187), *critical point* (p 188), *relative maximum* and relative minimum (p 189), *concave up* and *concave down* (p 202), *point of inflection* (p 208), *vertical asymptote* (p 221), and *horizontal asymptote* (p 222). Review Theorem 2 about relative extreme values (p 190), the First Derivative Test for Relative Extrema (p 192), the Second Derivative Test for Relative Extrema (p 204), and the Strategy for Sketching Graphs (p 226).

My outline for sketching graphs is a little different. You can follow it instead of the author's, if you choose.

Sketching Graphs

(a) Classify the function: Polynomial, Rational, or Other

(b) Find vertical and horizontal asymptotes (if the function is rational)

(c) Find $f'(x)$ and $f''(x)$, simplify, and factor (if possible).

(d) Use $f'(x)$ to find the critical points (if any), and where the graph is increasing or decreasing.

(e) Use $f''(x)$ to find the points of inflection (if any), and where the graph is concave up or down.

(f) Find the relative extreme points (if any). If the second derivative is relatively nice, use the second derivative test to check whether the points you found are relative maxima or relative minima. If the second derivative looks complicated, go back to the first derivative and use the first derivative test.

(g) Plot the critical points and points of inflection (if they exist), and then plot a few nearby points. (The goal is to use calculus to find the important parts of the graph, and then plot more points in these regions.) If there are asymptotes, sketch the lines. Finally, sketch the graph.

Review problems: page 179 #21–39; 2.9 #41; 3.1 #5, 11; 3.2 #12, 19, 22; 3.3 #7, 13, 15, 21, 36;