MATH 240 **TEST II** **October 17, 1997**

Prof. John Beachy

Show all of the work necessary to justify your answers.

1. (25 pts) Let A be the following matrix. $A = \begin{bmatrix} 1 & -2 & -3 & 0 & 3 \\ 0 & 1 & 2 & 0 & -2 \\ 3 & 1 & 5 & 1 & -5 \\ 1 & 1 & 3 & 1 & -3 \end{bmatrix}$

(a) Find the reduced row echelon form of A.
(b) Find the rank of A.
(c) Find a basis for the row space of A.
(d) Find a basis for the column space of A.
(e) Find a basis for the nullspace of A.

2. (10 pts) Determine whether the given sets of vectors are linearly dependent or linearly independent. Explain your answers.

(a) $\{(1, 3, -2), (-2, -6, 4)\}$ in $\mathbf{R}^3$
(b) $\{t^3, t^2 - 1, t^2 + 1, 4t, 2t - 3\}$ in P_3

3. (10 pts) Let W be the subspace of P_3 spanned by the vectors $t^2 - 1, t^2 + 1, 4t, 2t - 3$. Find a basis for W consisting of a subset of the given vectors.

4. (15 pts) Let $S = \left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$ and $T = \left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \end{bmatrix} \right\}$ be ordered bases for $\mathbf{R}^2$. Let $\mathbf{v} = \begin{bmatrix} 1 \\ 5 \end{bmatrix}$ and $\mathbf{w} = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$.

(a) Find the coordinate vectors $[\mathbf{v}]_T$ and $[\mathbf{w}]_T$ of $\mathbf{v}$ and $\mathbf{w}$ with respect to the basis T.
(b) Find the transition matrix $P_{S \leftarrow T}$.
(c) Use $P_{S \leftarrow T}$ to find the coordinate vectors of $\mathbf{v}$ and $\mathbf{w}$ with respect to the basis S.

5. (20 pts) Let M_{22} be the vector space of all 2×2 matrices, and let $Q = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$.

(a) Let W be the set of all matrices A in M_{22} such that $AQ = 0$. Show that W is a subspace of M_{22}.

(b) Find a basis for W, and find the dimension of W.

6. (10 pts) Let $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ be linearly independent vectors in a vector space V. Show that the set $\{\mathbf{v}_2, \mathbf{v}_3 - \mathbf{v}_2, \mathbf{v}_1 + \mathbf{v}_2 - \mathbf{v}_3\}$ is a linearly independent set.

7. (10 pts) Let $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ be column vectors in $\mathbf{R}^3$, and let A be a 3×3 matrix. Prove that if $\{A\mathbf{v}_1, A\mathbf{v}_2, A\mathbf{v}_3\}$ form a basis for $\mathbf{R}^3$, then $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ also form a basis for $\mathbf{R}^3$.