MATH 240 HOMEWORK 7 NAME ______________________

Prof. J. Beachy Wednesday, 11/13/02 Score / 20

1. Let $L : \mathbf{R}^3 \to \mathbf{R}^3$ be defined by $L\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} x_1 + x_2 \\ x_1 - x_2 \\ x_1 + x_3 \end{bmatrix}$, for all vectors in $\mathbf{R}^3$.

Let S be the standard basis for $\mathbf{R}^3$, and let T be the basis $\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \right\}$.

(a) Find the matrix representation $M_{S \leftarrow S}(L)$ of L with respect to S.

(b) Find the transition matrices $P_{T \leftarrow S}$ and $P_{S \leftarrow T}$.

(c) Find the matrix representation $M_{T \leftarrow T}(L)$ of L with respect to T directly,
and by computing $P_{T \leftarrow S} \cdot M_{S \leftarrow S}(L) \cdot P_{S \leftarrow T}$.

2. Let $L : \mathbf{R}^3 \to \mathbf{R}^3$ be defined by $L\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} x_3 \\ x_2 + x_3 \\ x_1 + x_2 + x_3 \end{bmatrix}$, for all vectors in $\mathbf{R}^3$.

Let S be the standard basis for $\mathbf{R}^3$, and let T be the basis $\left\{ \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}$.

(a) Find the matrix representation $M_{S \leftarrow S}(L)$ of L with respect to S.

(b) Find the transition matrices $P_{T \leftarrow S}$ and $P_{S \leftarrow T}$.

(c) Find the matrix representation $M_{T \leftarrow T}(L)$ of L with respect to T directly,
and by computing $P_{T \leftarrow S} \cdot M_{S \leftarrow S}(L) \cdot P_{S \leftarrow T}$.