

- (1.1, p21, #7) Let S be a nonempty finite set with a binary operation $*$ that satisfies the associative law. Show that S is a group if $a * b = a * c$ implies $b = c$ and $a * c = b * c$ implies $a = b$ for all $a, b, c \in S$. What can you say if S is infinite?
- (1.1, p21, #8) Prove that if G is a group and $a, b \in G$, then the equations $ax = b$ and $xa = b$ have unique solutions. Conversely, prove that if G is a nonempty set with an associative binary operation in which the equations $ax = b$ and $xa = b$ have solutions for all $a, b \in G$, then G is a group.
- (1.1, p21, #8) Let G be a set with an associative binary operation $*$. Prove that G is a group if (i) there exists a *left identity* $e \in G$ such that $e * a = a$ for each $a \in G$, and (ii) for each $a \in G$ there exists a *left inverse* $b \in G$ such that $b * a = e$.
- (1.1, p21, #20) Let F be a field with q elements. Find $|\text{GL}_n(F)|$.
- (1.2, p31, #13) Show that in $\text{SL}_2(\mathbf{Z}_3)$ the elements $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ and $\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$ generate a subgroup isomorphic to the quaternion group Q_8 .
- (1.2, p31, #20) Let G be a group with p^k elements, where p is a prime number and k is a positive integer. Prove that G has a subgroup of order p .