SOLVED PROBLEMS: SECTION 1.2

13. Check that any ring homomorphism preserves units, idempotent, and nilpotent elements.

14. Show that if R is any ring, then there is a unique ring homomorphism from $\mathbf{Z}$ into R.

15. Let R be a commutative ring, and let a be an element of R. The *annihilator* of a is $\mathrm{Ann}(a) = \{r \in R \mid ra = 0\}$. Prove that $\mathrm{Ann}(a)$ is an ideal of R.

16. Show that the matrix ring $\mathrm{M}_2(\mathbf{Q})$ is a simple ring.

17. A ring homomorphism $\phi : R \to S$ is called a ring *epimorphism* if for every pair of ring homomorphisms $\theta, \psi : S \to T$, $\theta\phi = \psi\phi$ implies $\theta = \psi$. Prove that any onto ring homomorphism is a ring epimorphism. Show that the inclusion mapping $\iota : \mathbf{Z} \to \mathbf{Q}$ is a ring epimorphism.

18. A ring homomorphism $\phi : R \to S$ is called a *ring monomorphism* if for every pair of ring homomorphisms $\theta, \psi : T \to R$, $\phi\theta = \phi\psi$ implies $\theta = \psi$. Prove that a ring homomorphism is a ring monomorphism if and only if it is one-to-one.

    Hint: If $\phi : R \to S$ is a ring monomorphism, use the subring of $R \oplus R$ defined by

    $T = \{(a, b) \mid a, b \in R \oplus R \text{ and } \phi(a) = \phi(b)\}$.

19. Let R be a commutative ring, let a be a unit of R, and let b be any element of R. Define a function $\phi : R[x] \to R[x]$ by $\phi(f(x)) = f(ax + b)$, for all $f(x) \in R[x]$. Show that ϕ is an automorphism of $R[x]$.