MATH 435 Spring 2009

Partial Solutions of Test 1 Review Problems

2. (a) $g(x) = 2\sin x$

   $g'(x)$ strictly decreases from 1 to -1 as x increases from $\frac{\pi}{3}$ to $\frac{2\pi}{3}$.

   (b) $g(x) = x + e^{-x} - \sin x$

   (c) $g(x) = \sqrt{2+x}$

3. Use Newton's Method.

5. (b) Change the variable $y = x - x_i$

   Then

$$\begin{aligned}\psi(x) &= (x - x_{i-1})(x - x_i)(x - x_{i+1}) \\ &= (y + h)y(y - h) \\ &= \psi(y)\end{aligned}$$

   To find max of $\psi(y)$ over $[-h, h]$, compute

$$\begin{aligned}\psi'(y) &= 3y^2 - h^2 \\ &= 0\end{aligned}$$

   gives $y = \pm\frac{h}{\sqrt{3}}$

   Hence $\max_{[x_{i-1},x_{i+1}]} |(x - x_{i-1})(x - x_i)(x - x_{i+1})| = \frac{2h^3}{3\sqrt{3}}$.

6. Maximum value of $\psi(x) = (x - x_0)(x - x_1)$ over $[x_0, x_1]$ occurs at $x = \frac{x_0+x_1}{2}$.

   Hence $\max_{x\in[x_0,x_1]} \psi(x) = \frac{(x_1-x_0)^2}{4}$.