Practice Problems for Exam #3

1. Suppose the first derivative of $y = f(x)$ is $y' = x^{-1/3}(x-1)$. Find all critical points, local extrema, and points of inflection. Also, determine where $f(x)$ is increasing, decreasing, concave up and concave down.

2. Sketch the graph of a function which satisfies all the following conditions:

$$y' > 0 \qquad 0 < x < 1,\ 2 < x < \infty$$

$$y' < 0 \qquad -\infty < x < 0,\ 1 < x < 2$$

$$y'' > 0 \qquad -\infty < x < 1,\ 1 < x < 3$$

$$y'' < 0 \qquad 3 < x < \infty$$

3. What are the dimensions of the lightest open-top right circular can that will hold a volume of 1000 cm^3? (In other words, minimize the surface area for such a can.)

4. Use Newton's method to approximate $\sqrt[3]{3}$. Start with $x_1 = 1$ and find x_3.

5. Find the most general antiderivative of the given function. Show all steps and check your answers by differentiation.

a) $f(x) = -7\sin x$

b) $f(x) = \frac{1-x^3}{x^5}$

c) $f(x) = 2x(x^{1/2} - x^{3/2})$

6. Estimate the area under the graph of $y = \sin(x)$ on $[0, \pi]$. Use four rectangles and left endpoints. Draw an appropriate picture. Repeat for right endpoints.

7. Find a general formula for the Riemann sum of x^4 on $[0, 2]$ using right endpoints. (You don't need to evalutate the sum or find the limit.)

8. Evaluate the definite integral $\displaystyle\int_{-1}^{3} 2 + |x|\, dx$.