

Newton was Limited (so to speak)

The quotes below are from Newton's book *Method of Fluxions*, which hit the streets in 1736. Of course, it was written in Latin (as was the fashion back then), so what appears below is really a translation. We start by looking at fluents, which are denoted by the variables x and y .

... and the velocities by which every fluent is increased by its generating motion (which I may call fluxions, or simply velocities) I shall represent by the same letters pointed, thus \dot{x} , \dot{y} .

Newton's infinitesimals are called "moments of fluxions," which are represented by $\dot{x}o$ and $\dot{y}o$, o being "an infinitely small quantity."

Thus, let any equation $x^3 - ax^2 + axy - y^3 = 0$ be given, and substitute $x + \dot{x}o$ for x , $y + \dot{y}o$ for y , and there will arise

$$\begin{aligned} x^3 + 3x^2\dot{x}o + 3x(\dot{x}o)^2 + (\dot{x}o)^3 - ax^2 - 2ax\dot{x}o - a(\dot{x}o)^2 + axy + ay\dot{x}o \\ + a\dot{x}o\dot{y}o + ax\dot{y}o - y^3 - 3y^2\dot{y}o - 3y(\dot{y}o)^2 - (\dot{y}o)^3 = 0. \end{aligned}$$

Now, by supposition, $x^3 - ax^2 + axy - y^3 = 0$, which therefore, being expunged and the remaining terms being divided by o , there will remain

$$3x^2\dot{x} + 3x\dot{x}^2o + \dot{x}^3o^2 - 2ax\dot{x} - a\dot{x}^2o + ay\dot{x} + a\dot{x}y + ax\dot{y} - 3y^2\dot{y} - 3y\dot{y}^2o - \dot{y}^3o^2 = 0.$$

But whereas o is supposed to be infinitely little, that it may represent the moments of quantities, the terms that are multiplied by it will be nothing in respect to the rest; I therefore reject them, and there remains

$$3x^2\dot{x} - 2ax\dot{x} + ay\dot{x} + ax\dot{y} - 3y^2\dot{y} = 0.$$

But wait Mr. Newton, you divided by o earlier and now you seem to say that $o = 0$! Did he actually divide by zero? If he did, is this okay or is everything wrong here? If he didn't, how can he arrive at the last equation above? Can you give some explanation for this?