

Math 420 Section 2

Final Exam

1. State the definition of a group. Give an example of an abelian group of order 8 which is not cyclic.
2. State the definition of isomorphism. Is D_4 isomorphic to \mathbb{Z}_8 ?
3. a) State the definition of congruence modulo m .
b) How many congruence classes are in \mathbb{Z}_{42}^\times ?
4. a) Find the greatest common divisor of 132465 and 634152.
b) Write 3 as a linear combination of 378 and 141.
5. What is the largest order of an element of \mathbb{Z}_{41}^\times ? Find an element of this order.
6. List the elements of D_5 which are even permutations.
7. The dihedral group D_4 is a subgroup of S_4 . Say two elements $\sigma, \tau \in S_4$ are equivalent if $\sigma \circ \tau^{-1} \in D_4$, i.e., if $\sigma \in D_4 \tau$. This is an equivalence relation (you need not prove it). How many equivalence classes are there, and what are the equivalence classes?
8. State Lagrange's theorem.
9. Prove that the intersection of two subgroups of a group is also a subgroup of that group.
10. Prove that a finite group of prime order is cyclic.