

Little Results from the
Axioms for Polynomials
(with the same old proof hints)

Lemma: The additive identity is unique.

To see this, try adding two putative additive identities together. What do you get?

Lemma: For any $P(X)$, the additive inverse of $P(X)$ is unique.

To see this, add $P(X)$ to two putative inverses, getting 0. Now add the first inverse to each sum. What do you get?

Lemma: For any polynomial $P(X)$, $P(X) \times 0 = 0$.

To see this, look at $(0 + 0) \times P(X)$.

Lemma: For any polynomial $P(X)$, $-1 \times P(X) = -P(X)$.

To see this, see what happens when you multiply $(1 + -1)$ by $P(X)$ using what we already know.

Lemma: If $P(X) \times Q(X) = 0$, then either $P(X) = 0$ or $Q(X) = 0$.

(Recall we had the following little result for integers: If $a > 0$, then $-a < 0$. If $a < 0$, then $-a > 0$.)

Does the old “hint” still work?

“To see this, show that $a \times b = 0$ implies that $a \times b = a \times (-b) = -(a \times b)$.”

Hmmmmmm.....