

## Axioms for Polynomials

The set of polynomials with rational coefficients,  $\mathbb{Q}[X]$ , is a non-empty set with a binary operations  $+$  and  $\times$ . It has a shockingly familiar set of axioms.

The addition  $+$  satisfies the following:

1.  $(a + b) + c = a + (b + c)$  for any  $a, b, c \in \mathbb{Q}[X]$  ( $+$  is associative);
2. there is a polynomial  $0$  where  $0 + a = a + 0 = a$  for all  $a \in \mathbb{Q}[X]$  (there is an additive identity);
3. for every  $a \in \mathbb{Q}[X]$  there is a  $b \in \mathbb{Q}[X]$  where  $a + b = b + a = 0$  (every element has an additive inverse); and
4.  $a + b = b + a$  for all  $a, b \in \mathbb{Q}[X]$  (addition is commutative).

The multiplication  $\times$  satisfies the following:

1. it is associative;
2. it has an identity not equal to the additive identity;
3. it distributes through addition on both the left and right, i.e.,  $a \times (b + c) = a \times b + a \times c$  and  $(a + b) \times c = a \times c + b \times c$  for all  $a, b, c \in \mathbb{Q}[X]$ ; and
4. it is commutative.

Is there is an order relation on  $\mathbb{Q}[X]$  which totally orders  $\mathbb{Q}[X]$  in the same way  $\leq$  totally orders  $\mathbb{Z}$ ?