THE FANO PLANE

The Fano plane is a collection of 7 points

\[ P = \{A, B, C, D, E, F, G\} \]

and 7 lines, determined by the following drawing:

The seven lines are

1. \( A B D \)
2. \( A C E \)
3. \( B C F \)
4. \( A F G \)
5. \( B E G \)
6. \( C D G \)
7. \( E D F \)

The Fano plane \( P \) is also called the **projective plane** of order 2 and the **Steiner triple system** on 7 points.

**Prove:** No matter how you color the seven points in the Fano plane \( P \) using two colors (red and blue), it will always be the case that at least one of the seven lines will contain three points all of the same color.