1. Compute the volume of the solid obtained by revolving the region in the first quadrant bounded by the curve $y = 1 - x^2$ about the line $x = -2$.

A thin vertically oriented ("skinny side down") rectangle revolved about the line $x = -2$ will generate a cylindrical shell. The height of the shell is the height of the rectangle, which is the distance from the $x$-axis up to the curve: $1 - x^2$. The radius of the shell is the distance from the slice to the axis of revolution. If the slice is located horizontally at $x$, then this distance is $x - (-2) = x + 2$. Note that $0 \leq x \leq 1$ in this region. Thus, the volume of the solid is

$$
\int_0^1 2\pi(1 - x^2)(x + 2) \, dx = 2\pi \int_0^1 2 + x - 2x^2 - x^3 \, dx
$$
$$
= 2\pi \left( 2x + \frac{x^2}{2} - \frac{2x^3}{3} - \frac{x^4}{4} \right) \bigg|_0^1
$$
$$
= 2\pi \left( 2 + \frac{1}{2} - \frac{2}{3} - \frac{1}{4} \right) = \frac{19\pi}{6}.
$$

One may also use horizontally oriented rectangles. In order to carry this out, we note that the curve $y = 1 - x^2$ in the first quadrant may be written $x = \sqrt{1 - y}$. A thin horizontally oriented rectangle will generate a washer. The inner radius of the washer is the distance from the axis of revolution to the left end of the rectangle: $0 - (-2) = 2$. The outer radius is the distance from the axis of revolution to the right end of the rectangle: $\sqrt{1 - y} - (-2) = \sqrt{1 - y} + 2$. Note that $0 \leq y \leq 1$ in this region. Thus, the volume of the solid is

$$
\int_0^1 \pi((\sqrt{1 - y} + 2)^2 - 2^2) \, dy = \pi \int_0^1 1 - y + 4\sqrt{1 - y} \, dy
$$
$$
= \pi \int_0^1 (u + 4\sqrt{u})(-1) \, du
$$
$$
= \pi \int_0^1 u + 4\sqrt{u} \, du
$$
$$
= \pi \left( \frac{u^2}{2} + \frac{4u^{3/2}}{3/2} \right) \bigg|_0^1
$$
$$
= \pi \left( \frac{1}{2} + \frac{8}{3} \right) = \frac{19\pi}{6},
$$

where we have used the substitution $u = 1 - y$. \[1\]
2. Compute the volume of the solid obtained by revolving the region above about the $x$-axis.

A thin vertically oriented rectangle revolved about the $x$-axis now generates a disk of radius $1 - x^2$. Thus, the volume of the solid is

$$
\pi \int_0^1 (1 - x^2)^2 \, dx = \pi \int_0^1 1 - 2x^2 + x^4 \, dx
$$

$$
= \pi \left( x - \frac{2x^3}{3} + \frac{x^5}{5} \right) \bigg|_{x=0}^{1}
$$

$$
= \pi \left( 1 - \frac{2}{3} + \frac{1}{5} \right) = \frac{8\pi}{15}.
$$

A thin horizontally oriented rectangle revolved about the $x$-axis now generates a cylindrical shell of height $\sqrt{1 - y}$ and radius $y$, so the volume of the solid is

$$
2\pi \int_0^1 \sqrt{1 - y} \cdot y \, dy = 2\pi \int_0^1 \sqrt{u}(1 - u)(-1) \, du
$$

$$
= 2\pi \int_0^1 u^{1/2} - u^{3/2} \, du
$$

$$
= 2\pi \left( \frac{u^{3/2}}{3/2} - \frac{u^{5/2}}{5/2} \right) \bigg|_{u=0}^{1}
$$

$$
= 2\pi \left( \frac{2}{3} - \frac{2}{5} \right) = \frac{8\pi}{15},
$$

where again we have used the substitution $u = 1 - y$. 