1. Compute the volume of a pyramid with an equilateral triangle base of side length 200 meters and a height of 100 meters.

A front view of this pyramid will be a triangle of height 100 and base 200 (both in meters). Plot this triangle in the $xy$-plane with the base on the $x$-axis and the top vertex at the point $(0, 100)$. The sides are segments of the lines given by $x + y = 100$ and $x - y = -100$. Thus, as a function of $y$ (for $0 \leq y \leq 100$), the distance from one side of this triangle to the other is

$$100 - y - (-100 + y) = 200 - 2y.$$  

(You can check that this is correct when $y = 0$ and when $y = 100$.) A representative “slice” of our pyramid at an elevation $y$ is an equilateral triangle with side length $200 - 2y$. The general formula for the area of an equilateral triangle of side length $s$ is $s^2\sqrt{3}/4$, thus the area of our representative slice is

$$A(y) = (200 - 2y)^2 \sqrt{3}/4 = \sqrt{3}(100 - y)^2.$$  

Integrating this cross-sectional area will give us the volume:

$$\int_0^{100} \sqrt{3}(100 - y)^2 \, dy = \sqrt{3} \int_0^{100} (100 - y)^2 \, dy$$

$$= \sqrt{3} \int_0^{100} u^2(-1) \, du$$

$$= \sqrt{3} \int_0^{100} u^2 \, du$$

$$= \sqrt{3} \left[ \frac{u^3}{3} \right]_{u=0}^{u=100}$$

$$= \frac{100^3}{\sqrt{3}}.$$  

Here we used the substitution $u = 100 - y$. Also, this volume is in cubic meters.

2. Compute the volume of a right circular cone with base diameter 200 meters and height 100 meters.

This is very similar to the example above. A front view of this cone is exactly the same as the front view of the pyramid above, giving us the same triangle. Now a representative “slice” of our cone is a circle (instead of an equilateral triangle) of diameter $200 - 2y$, so radius $100 - y$. Since the general formula for the area of a circle of radius $r$ is $\pi r^2$, the area
of our representative slice at elevation $y$ is

$$A(y) = \pi (100 - y)^2.$$ 

Integrating this cross-sectional area will give us the volume. Since this area is just $\pi/\sqrt{3}$ times the area in the first example, the volume here will just be

$$\frac{\pi}{\sqrt{3}} \times \frac{100^3}{\sqrt{3}} = \frac{\pi 100^3}{3}.$$