1. Suppose $\sum a_n$ and $\sum b_n$ are series of positive terms and $a_n \leq b_n$ always.
   a) If $\sum a_n$ converges, what can you say about $\sum b_n$?
   b) If $\sum b_n$ converges, what can you say about $\sum a_n$?
   c) If $\sum a_n$ diverges, what can you say about $\sum b_n$?
   d) If $\sum b_n$ diverges, what can you say about $\sum a_n$?

2-11. Determine whether the following series converge or diverge.
   2. $\sum_{n=1}^{\infty} \frac{n}{2n^3 + 1}$
   3. $\sum_{n=1}^{\infty} \frac{n^4}{n^5 + 1}$
   4. $\sum_{n=1}^{\infty} \frac{5^n}{1 + 6^n}$
   5. $\sum_{t=2}^{\infty} \frac{\ln t}{t}$
   6. $\sum_{n=1}^{\infty} \frac{2}{\sqrt[4]{4n^5} + 3}$
   7. $\sum_{n=2}^{\infty} \frac{1}{\sqrt{n^2 - 1}}$
   8. $\sum_{n=1}^{\infty} \frac{e^{1/n}}{n}$
   9. $\sum_{n=1}^{\infty} \frac{n^{10}}{n!}$
   10. $\sum_{n=1}^{\infty} \frac{n!}{n^n}$
   11. $\sum_{n=1}^{\infty} \frac{\sin^2(n)}{n^{3/2}}$

12. Use the sum of the first five terms to approximate the sum of the series $\sum_{n=1}^{\infty} \frac{1}{3^n + 5^n}$. Estimate the error.

13. Use the sum of the first ten terms to approximate the sum of the series $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^4 + 1}}$. Estimate the error.

14. Suppose $\sum a_n$ is a series with positive terms that diverges. Is there a divergent series $\sum b_n$ of positive terms with $b_n < a_n$ always? Is there a “smallest” divergent series of positive terms?

15. Suppose $\sum a_n$ is a series with positive terms that converges. Is there a convergent series $\sum b_n$ of positive terms with $b_n > a_n$ always? Is there a “largest” convergent series of positive terms?
16. The *Cauchy Condensation Test* is a useful way to see whether certain series converge. Suppose $\sum a_n$ is a series of positive terms where the terms (eventually) decrease to zero: $a_{n+1} \leq a_n$ for all $n$ sufficiently large and $a_n \to 0$ as $n \to \infty$. Then the series $\sum a_n$ converges if and only if the related series $\sum 2^na_{2^n}$ converges.

a) Show that the harmonic series $\sum 1/n$ diverges using the Cauchy Condensation Test.

b) What does the Cauchy Condensation Test say about the $p$-series $\sum 1/n^p$ for various values of $p$?

c) Use the Cauchy Condensation Test to show that the series $\sum_{t=2}^{\infty} \frac{1}{t \ln t}$ diverges.