1. Evaluate the integral $\int x \ln x \, dx$ by parts, using $u = \ln x$ and $dv = x \, dx$.

2. Evaluate the integral $\int \theta \sin \theta \, d\theta$ by parts, using $u = \theta$ and $dv = \sin \theta \, d\theta$.

3. Evaluate the integral $\int \arctan x \, dx$ by parts, using $u = \arctan x$ and $dv = \, dx$.

4-9. Evaluate the integral.
4. $\int t^2 \exp(t) \, dt$
5. $\int_0^1 \sin^{-1} x \, dx$
6. $\int_0^\theta \theta \cos(\pi \theta) \, d\theta$
7. $\int (\ln x)^2 \, dx$
8. $\int_0^\pi \sin^2 x \, dx$
9. $\int \sec^3 \theta \, d\theta$

10-14. The following formulas may be found in any typical table of integrals (like the one in the back of your textbook, for example). Use integration by parts to verify these formulas.

10. $\int (\ln x)^n \, dx = x(\ln x)^n - n \int (\ln x)^{n-1} \, dx$
11. $\int u^n e^{au} \, du = \frac{1}{a} u^n e^{au} - \frac{n}{a} \int u^{n-1} e^{au} \, du$
12. $\int e^{au} \sin(bu) \, du = \frac{e^{au}}{a^2 + b^2} (a \sin(bu) - b \cos(bu)) + C$
13. $\int y^n \ln y \, dy = y^{n+1} \ln y \frac{n+1}{(n+1)^2} + C \quad n \neq -1$
14. $\int \cos^n \theta \, d\theta = \frac{1}{n} \cos^{n-1} \theta \sin \theta + \frac{n-1}{n} \int \cos^{n-2} \theta \, d\theta, \quad n \geq 1$

15. Sketch the graph of $y = \sin x$, $0 \leq x \leq \pi/2$. Note that this is also the graph of $x = \sin^{-1} y$, $0 \leq y \leq 1$. You see that this graph separates the rectangle $0 \leq x \leq \pi/2$, $0 \leq y \leq 1$ into two pieces. The total area of this rectangle is $\pi/2$. Explain why the areas of the two pieces are equal to $\int_0^{\pi/2} \sin x \, dx$ and $\int_0^1 \sin^{-1} y \, dy$. Conclude that

$$\int_0^{\pi/2} \sin x \, dx + \int_0^1 \sin^{-1} y \, dy = \frac{\pi}{2}.$$  

Now evaluate $\int_0^{\pi/2} \sin x \, dx$ and use that to find $\int_0^1 \sin^{-1} y \, dy$. Your answer should agree with exercise #5 above.