MATH 230, Spring 2012 Homework
Partial Fractions

1-8. Compute the partial fraction decomposition of the following rational functions.

1. \frac{5x - 13}{(x - 3)(x - 2)}
2. \frac{5x - 7}{x^2 - 3x + 2}
3. \frac{x + 4}{(x + 1)^2}
4. \frac{x + 1}{x^2(x - 1)}
5. \frac{x^2 + 8}{x^2 - 5x + 6}
6. \frac{x}{x^3 - x^2 - 6x}
7. \frac{5x^2 + 2x + 1}{(x + 1)(x^2 + 1)}
8. \frac{x - 2}{x^3 + 2x}

9-14. Evaluate the following integrals.

9. \int \frac{dx}{x^2 + 2x}
10. \int_{1/2}^{1} \frac{y + 4}{y^2 + y} \, dy
11. \int_{-1}^{0} \frac{t^3}{t^2 - 2t + 1} \, dt
12. \int_{1}^{\sqrt{3}} \frac{3r^2 + r + 4}{r^3 + r} \, dr
13. \int \frac{s^4 + 81}{s(s^2 + 9)^2} \, ds
14. \int \frac{e^x}{e^{2x} + 3e^x + 2} \, dx

When we say any rational function can be integrated, we make two assumptions: we are assuming that any polynomial (the denominator) can be factored completely and also that the “answer” one gets after performing the partial fraction decomposition can be integrated. The next few exercises address this second assumption.

15. Find formulas for \int \frac{dx}{x + a}, \int \frac{dx}{(x + a)^2} and \int \frac{dx}{(x + a)^3}. In general, for a positive integer n, what is \int \frac{dx}{(x + a)^n}? (Note that n = 1 is a special case.)

16. Find a general formula for \int \frac{x \, dx}{(x^2 + a^2)^n} for any positive integer n. (Hint: n = 1 is a special case.)
17. Find formulas for \( \int \frac{dx}{(x^2 + a^2)^2} \) and \( \int \frac{dx}{(x^2 + a^2)^3} \). What can you say about the general case for \( \int \frac{dx}{(x^2 + a^2)^n} \) for any positive integer \( n \)?

If you use the substitution \( z = \tan(x/2) \), then

\[
dx = \frac{2dz}{1 + z^2}, \quad \cos x = \frac{1 - z^2}{1 + z^2}, \quad \sin x = \frac{2z}{1 + z^2}.
\]

Use this substitution to evaluate the following integrals.

18. \( \int_{0}^{\pi/2} \frac{dx}{1 + \sin x} \).

19. \( \int_{\pi/2}^{2\pi/3} \frac{\cos x}{\sin x \cos x + \sin x} \, dx \).